Magnetic Circuit Units

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Synopsis.—*A proposition to introduce a change in the units of the magnetic circuit was made at the meeting of the International Electrotechnical Commission (I. E. C.) in Bellagio, September 1927. The proposition was referred to an international subcommittee for consideration and report, by the Advisory I. E. C. Committee on Nomenclature. The subcommittee has recently reported, recommending that the matter be placed on the agenda of the next I. E. C. meeting, and that in the meanwhile, the subject should be discussed in all the national committees, more especially in the various electrical engineering societies, so as to pave the way for well considered international action. This paper has been prepared in the hope of presenting the questions definitely, with a view to eliciting discussion and opinions that may be helpful from the international point of view.*

As an aid to the formulation of views on the subject among those who may not have given special attention to the historical development of the magnetic-circuit units, a short outline of their history is here submitted.

Recent International Proposals. At the meeting of the International Electrotechnical Commission (I. E. C.) at Bellagio in September 1927, the Italian delegation submitted† a proposal to establish a new international unit of magnetic flux ϕ in the ‡"practical system," with a magnitude of 10^8 C. G. S. magnetic units, or corresponding to what is sometimes called a "volt-second," and to assign to this new unit of flux the name "maxwell."

After some discussion, the matter was referred to an international subcommittee of seven, (representing the national I. E. C. committees of France, Germany, Great Britain, Holland, Italy, Russia, and the United States) for consideration and report to the I. E. C. Committee No. 1 on *Nomenclature.*

The subcommittee discussed the matter by correspondence distributed through the General Secretary's office in London, during 1928 and 1929. It presented a report in September 1929 to Dr. Mailloux, the president of the I. E. C. Nomenclature Committee. A copy of the report is appended to this paper.

Need for General Consideration and Discussion. From the correspondence among the members of the international subcommittee, it was evident that although no official change has been made in the international status of magnetic-circuit units since the Paris International Electrical Congress of 1900, yet there exists a considerable difference of usage of magnetic units in the various countries, not merely as to the names, but also as to the definitions of the units. It is therefore in the interest of all countries that the whole question of the magnetic units and their names should be considered afresh, with a view to arriving at a sound, simple, and satisfactory international agreement upon the same. While, therefore, the subcommittee could not endorse the proposal to change the value of the *maxwell* from 1 C. G. S. to 10^8 C. G. S. units of magnetic flux ϕ, it recommends that the matter be discussed in all the twenty-five countries participating in the I. E. C., particularly among their engineering societies, in order to elicit useful suggestions and to pave the way for well considered international action. This paper aims to present a brief history of the development of the magnetic-circuit units, especially during the last forty years, in so far as may be helpful in visualizing the present status of these units, to indicate the reasons existing for changing that status, and of submitting a few suggestions in that direction by way of starting the discussion in the American Institute of Electrical Engineers.

Outline History of the Magnetic-Circuit Units. The first analysis of the behavior of a suspended magnetic needle appears to have been given by Ampère in 1820.* The first analysis referred to absolute measure, *i. e.*, to units of length, mass, and time, is attributed to Gauss† in 1833. In 1840, W. E. Weber, who collaborated with Gauss, showed how to measure the strength of a steady electric current in absolute measure with the aid of a tangent galvanometer, and later, with an electrodynamometer. The magnetic units employed by Gauss and Weber were in the millimeter-milligram-second system (mg-mm-sec.) or M. M. S. system, in direct decimal relation with the international metric system.

In 1861, two electric telegraph engineers, Latimer Clark and Charles Bright, read a paper at the Manchester meeting of the British Association for the Advancement of Science (generally known as the B. A.) calling attention to the needs for scientific electrical units, standards, and measures in telegraph engineering. This led to the formation by the B. A., in that year, of a committee‡ on "Standards of

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†Bibliography 29, pp. 121-123. Report of I. E. C. Meeting at Bellagio.

‡The term "practical system" as here used, refers to the series of international electrical units, substantially as adopted at the Electrical International Congress of Chicago in 1893; *i. e.*, the international volt, ohm, ampere, etc.

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*Ampère, André M., "Sur l'état magnetique des corps qui transmettent un courant d'électricité," *Ann. Chem. Phys.*, XVI, Paris, 1821.

†Gauss, Karl Friedrich, "Intensitas vis magneticae terrestris ad mensuram absolutam revocata," Gottingen, 1833.

‡British Association Reports. "On Electrical Standards," 1861, 1863, 1864, 1865, 1867. (First B. A. Committee.)

Electrical Resistance." Its members were eminent in basic and applied science. This first B. A. committee set to work to establish units and standards of resistance, e. m. f., and current in absolute measure, making reports to the B. A. from 1863 to 1867, when it was reorganized and the second B. A. committee§ was appointed "for the Selection and Nomenclature of Dynamical and Electrical Units." The first B. A. committee issued standard B. A. ohms and microfarads in the meter-gram-second* (M. G. S.) system, the B. A. standard ohm being made to represent 10^7 absolute M. G. S. units of resistance, and the B. A. standard microfarad to represent 10^{-13} M. G. S. units of capacitance. At the present date, these would be taken respectively as representing 10^9 C. G. S. units of resistance and 10^{-15} C. G. S. units of capacitance. The same B. A. standard condenser was originally called a *farad;** but by common consent its name was changed to *microfarad,* to preserve the unitary relations between the newly adopted practical units. The *volt* of 10^5 M. G. S. units of electromotive force was also recognized and adopted by the committee, as being not far from the e. m. f. of a Daniell zinc-copper cell, at that time well known in telegraphy; but so greatly did different voltaic cells of the same type vary in e. m. f., that it was not until 1873, that the first Clark zinc-mercury cell was developed as a working standard of e. m. f.

When the second B. A. Committee made its first report in 1873, it recommended changing the basis of the absolute units from the M. G. S. to the C. G. S. system,† mainly for the reason that in the latter, the systematic unit of density was the gram per cubic centimeter and substantially equal to the density of pure water; whereas in the former, the systematic unit of density was the gram per cubic meter or one millionth that of water. Thereafter, the M. G. S. system was superseded by the C. G. S. system, not only in electrical and magnetic units, but also in all branches of science. The change seems, however, to have been made against some opposition, since one member of the second committee went on record† as being opposed to the substitution of the centimeter for the meter.

The first International Congress of Electricians assembled at Paris, in 1881. It adopted the C. G. S. system as the basis of absolute units. It likewise adopted the *ohm, volt, ampere, coulomb,* and *farad* just as they are today, except for minor corrections, and all decimally derived from the C. G. S. system.

The second International Electrical Congress of 1889, also at Paris, adopted three more units in the "practical" system—the *joule, watt,* and *quadrant* (now the *henry*). The Congress also discussed magnetic units in the "practical" system, recommending the *weber* as the name of a practical unit of magnetic flux, defined as 10^8 C. G. S. units of magnetic flux ϕ, and also the *gauss* as the name for the practical unit of field intensity*, or 10^8 C. G. S. units of magnetic flux density B. No definite action, however, was taken by the Congress on either of these units.

The rapid development of dynamo-electric machines in the eighties, led to a demand among electrical engineers for magnetic units with corresponding names, to be used in the analysis and description of magnetic circuits.

In 1891, the A. I. E. E. appointed a committee on "Units and Standards" with especial reference to the study of magnetic-circuit units. The committee reported[4] in June 1891, recommending units of magnetomotive force, magnetic flux, magnetic induction density, and magnetic reluctance, all in the "practical" system, except, as was pointed out in the discussion, that the flux density was referred to the square centimeter, and was therefore a departure from the "practical" system. No names were offered for these units, but it was suggested that the A. I. E. E. should seek to have these units adopted at the next following international electrical congress, with names for the same. No action was taken on the report beyond accepting it and printing it in the Proceedings.

The subject of magnetic units was discussed at the third International Congress, held at Frankfort, in September 1891. The names *gauss* and *weber* were recommended, in the "practical" system, for the units of field intensity H, and magnetic flux ϕ, respectively, but no action was taken.

International Congress of Chicago. At the fourth International Electrical Congress, held at Chicago in 1893, the subject of magnetic units received attention in the Chamber of Deputies. The recommendation was recorded[5] that the C. G. S. system should be used for magnetic units, and that for the present no names should be given to them.

It is to be remembered that magnetic observatories had come into existence about the year 1850, for measuring and recording the earth's magnetic field in their respective localities. At first, these observatories used M. M. S. (mm-mg-sec.) units, but after about 1875 the records were kept in C. G. S. units. Among magneticians, this practise was regarded as a reason for keeping the magnetic units in the C. G. S. system. Many objections appeared in electrotechnical literature, between 1890 and 1894, to the adoption of magnetic units in the "practical" system. Up to that time, however, only the two names *dyne* and *erg* had been adopted for units in the C. G. S. system. All of the names for electric units and standards had been assigned by international congresses in the "practical" system. The retention of magnetic units in the C. G. S. system thus raised difficulties in the way of giving names to them. On the other hand, electrical engineers working with the design and construction of dynamo-electric machines, and with the analysis of their magnetic circuits, were insistent for unit names as well as unit magnitudes for the essential elements of those circuits.

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§First Report of the B. A. "Committee for the Selection and Nomenclature of Dynamical and Electrical Units," (Second B. A. Committee) 1873.

Second Report of same Committee, 1874. See also Appendix to Everett's "Illustration of the C. G. S. System of Units," 1875-1891.

*Clark, Latimer and Sabine, Robert, "Electrical Tables and Formulae," E. & F. N. Spon, London, 1871, pages 1-5 and Preface.

†Everett, J. D., "Illustrations of the C. G. S. System of Units," Macmillan & Co., 1891, Appendix.

*Although the term "field intensity" suggests magnetizing force H, yet the context seems to indicate that the *gauss* was intended for the unit of flux density B.

4. For numbered references see Bibliography.

In view of the recommendations of the Chicago Congress for retaining the working units of the magnetic circuit in the C. G. S. system, the Committee on "Units and Standards" of the A. I. E. E. reported to that body in March 1894* in favor of provisionally adopting the working units of the magnetic circuit in the C. G. S. magnetic system with the following names: (1) the *gilbert* as the C. G. S. unit of m. m. f. with the value $10/4\,\pi$ ampere-turn; (2) the *weber* as the C. G. S. unit of magnetic flux; (3) the *oersted* for the C. G. S. unit of reluctance; and (4) the *gauss* as the C. G. S. unit of flux density, or one *weber* per normal sq. cm. The report was discussed at two meetings of the Institute and provisionally adopted at the second.

In 1895, the Committee of Electrical Standards† of the British Association recommended the tentative adoption of the following units and names:

(1) As the unit of magnetic flux, 10^8 C. G. S. magnetic units, to be known as the *weber*.

(2) As the unit of magnetomotive force, the C. G. S. magnetic unit of m. m. f., to be known as the *gauss*.

In May 1900,‡ in view of the then approaching International Electrical Congress at Paris in August 1900, the A. I. E. E. Committee on Units and Standards reported to that body in favor of fixing the working units of the magnetic circuit in the C. G. S. system, and also of considering their rationalization (the elimination of the factor $4\,\pi$ from the unit of m. m. f.). The A. I. E. E. adopted the report and instructed its delegates to recommend accordingly at the Paris Congress.

Fifth International Electrical Congress, at Paris. At the Paris Congress, there was much difference of opinion and also some misunderstanding. It was generally agreed that the working units of the magnetic circuit should be retained in the C. G. S. system, but there was considerable opposition to giving them names. The Congress finally voted and printed in its report[7] the adoption of two names, *viz.*, the *gauss* for the C. G. S. unit of field intensity H, and the *maxwell* for the C. G. S. unit of magnetic flux ϕ. These two working magnetic units in the C. G. S. system were adopted at the close of an animated debate. The A. I. E. E. delegates supposed that the name *gauss* had been assigned by the Chamber to the C. G. S. unit of flux density B, and so reported to the A. I. E. E.‡ It was not until after the publication of the official Paris Congress report that the discrepancy was discovered.

International Conditions Subsequent to the Paris Congress of 1900. There has been no official international action on magnetic units since 1900; but usage in electrotechnical literature has varied considerably as between different countries. Indeed different writers in one and the same country have sometimes differed in their use of magnetic units. In America, the custom has been to use the *gilbert* for the working unit of magnetic potential, the *gilbert per centimeter* for the unit of magnetizing force or field intensity H, the *maxwell* for the unit of magnetic flux ϕ, the *gauss* for the unit of flux density B, and the *oersted* for the unit of reluctance; *i. e.*, for the reluctance of 1 cubic centimeter of free space; all in the C. G. S. system.[27] It has been recognized that the *gilbert* and *oersted* had no international status and had never been internationally adopted. In addition, however, to these more or less standardized working C. G. S. magnetic units, American electrotechnical literature contains various other magnetic units; such as the *ampere-turn* for m. m. f., the *line* for the C. G. S. unit of magnetic flux, and the *line per square inch* or *maxwell per square inch* as hybrid units of flux density B.

In European literature, the *gauss* has been used by some writers as the C. G. S. unit of magnetizing force or magnetic field H, and sometimes as the C. G. S. unit of magnetic flux density B. Some writers have also used the *gauss* indifferently for both H and B. As regards the *maxwell*, the usage has been more generally standardized, and where it has been employed, it has meant the C. G. S. unit of magnetic flux ϕ in nearly all cases. In some countries, however, the *maxwell* has been little used, being sometimes replaced by the *line*. The *gilbert* and the *oersted* have been rarely used outside of the United States.

In addition to the lack of complete international standardization of magnetic-unit terminology, there has also been manifest, in literature, a lack of complete standardization in the definitions of the magnetic quantities to which the units are applied. That is to say, there are differences of opinion as to the "dimensions" of magnetic units, and especially as to the dimensions of magnetic permeability μ. This divergence of opinion exists to some extent in all countries, and to a considerable extent in some countries. The question is rendered more difficult by the fact that, as was pointed out many years ago by Rücker,[2] it is not yet possible to assign dynamical dimensions, (in L, M, and T) to all electromagnetic quantities; so that we are unable at present to assign their precise dimensions. It is customary to express the dimensions of the various electromagnetic quantities either in terms of length, mass, time, and permeability ($L\ M\ T\ \mu$) or in terms of length, mass, time, and electric permittivity ($L\ M\ T\ \kappa$). This is equivalent to the admission that the dimensions of μ and κ are still undetermined, except that their geometrical mean $\sqrt{\mu\ \kappa}$ is the reciprocal of a propagation velocity, or $1/v$. If, however, the dimensions of μ are zero, so that μ is a mere numeric, it must follow that in the ordinary formula connecting magnetic force with magnetic flux density; *i. e.*, $B = \mu\, H$, the unit for B must have the same nature as the unit of H, and the same name will be applicable to both. If one of these quantities is expressed in *gausses*, the other must likewise be expressible in *gausses*. If, on the contrary, μ has physical dimensions, and is not a mere number, then B and H cannot properly be expressed in terms of the same unit. The question turns upon the definitions that are used for H and B. It is generally admitted that under certain definitions of B and H, their ratio μ will be a mere numeric, while under other definitions, their ratio will have physical dimensions. In order, therefore, to secure satisfactory international standardization of the C. G. S. magnetic units, it seems first necessary to reach international agreement on the underlying definitions of the quantities involved.

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*A. I. E. E. Trans., Vol. XI, 1894; Jan. 17, pp. 48-52; Mar. 21, pp. 124-132.

†"Electric Units and Standards" Circular No. 60 of the Bureau of Standards, Washington.

‡A. I. E. E. Trans., 1900, Vol. XVII, pp. 309-318, May 17, 1900.

[7] A. I. E. E. Trans., Vol. XVII, pp. 543-547, Nov. 21, 1900 and Bibliography 6.

In 1911,* the A. I. E. E. recommended the use of the following magnetic units, all in the C. G. S. magnetic system: the *gilbert-per-cm.* for magnetic force H, the *maxwell* for magnetic flux ϕ, and the *gauss* for magnetic density B. In the A. I. E. E. Standardization Rules for 1914† following the vote of the Paris 1900 Congress, the *gauss* is inserted as an alternative C. G. S. unit of magnetic intensity H, without withdrawing the same name from the unit of flux density. A footnote states "The *gauss* is provisionally accepted for the present, as the name of both the unit of field intensity and flux density, on the assumption that permeability is a simple numeric."

Magnetic Units in "Practical" Systems. It was pointed out by Maxwell that‡ the "practical" system of units forms an absolute system in which the unit of length is a quadrant of the earth, or 10^7 meters, the unit of mass an eleventh-gram or 10^{-11} g., and the unit of time the mean solar second. The discovery of this relation theoretically placed the "practical" units upon an independent basis, quite detached from the C. G. S. units. Unfortunately, the underlying unit of mass was so small and that of length so large, that the systematic use of the independent "practical" system became very inconvenient. Thus, with the *ampere* as unit of current, the logical and systematic unit of current density becomes in the Q. E. S. (*quadrant, eleventh-gram, second*) system the *ampere per square quadrant*, or 10^{-14} *amperes* per square meter. If the practical unit of density were arbitrarily taken as the *ampere per sq. m.*, or the *ampere per sq. cm.*, this would be adopting a hybrid unit outside of the Q. E. S. system, and the simplicity inherent in a logical absolute system would be lost, through the necessity of introducing numerical coefficients like 10^{-14} into computations involving current density. Consequently, although the Q. E. S. system of independent absolute units including the "practical" units, has been recognized as a "comprehensive" system for nearly fifty years, no writer has espoused the cause of magnetic units in the Q. E. S. system. The unit of magnetic field intensity H would presumably be the ampere-turn per quadrant, and the unit of flux density B would be referred to unit flux per square quadrant.

Giorgi System of M K S Ω Units. Commencing in 1901,* Giorgi pointed out that a rationalized absolute system of units could be used, including all the practical or volt-ampere-ohm series, by taking the meter as the unit of length, the kilogram as the unit of mass, the second as the unit of time, and the international ohm. Whereas the C. G. S. system was essentially dual, with a magnetic series and an electric series, interconnected by awkward coefficients of v or v^2 or their reciprocals, the M. K. S. Ω system was essentially single, or had only one series. Such a system may be described as a "unified" system. The M. K. S. Ω system has been favorably commented upon† by specialists in various countries. It takes the permeability of free space μ_0 as $4\ \pi \times 10^{-7}$, instead of unity as in the C. G. S. magnetic system, and it takes the specific permeability of steel or other magnetic medium μ, as a simple numeric. It is open to the objection that whereas the density of pure water in the C. G. S. system is 1 g. per cu. cm. and therefore unity, under proper specifications, the density of water in the M. K. S. Ω system is one thousand kilograms per cubic meter, or unit density is one thousandth that of water. So far as is known, no electrotechnical book has yet been written in the M. K. S. Ω system.

The "International" or C. G. S. S. System.[22, 23, 24] In 1916, Dellinger and Bennett suggested the "International" comprehensive unified system, including all the "practical" units, based on the centimeter as unit of length, the gram-seven or 10^7 gm., as unit of mass and the second as unit of time. This is sometimes called the centimeter-gram-seven-second or C. G. S. S. system. Like the Giorgi system, it is rationalized, and the ampere-turn becomes the rational and practical unit of

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*A. I. E. E. TRANS., 1911. Part III, Vol. XXX, Standardization Rules, p. 2564.

‡Bibliography 1. Chapter X. Dimensions of Units.

†A. I. E. E. TRANS., 1914. Part II, Vol. XXXIII, Standardization Rules, p. 1798.

*Bibliography 8, 9, 10, 12, 13, 14, 17.

†Bibliography 11, 15, 16, 19, 21, 23, 25.

m. m. f. in both. It takes the permeability of free space μ_0 as $4\ \pi \times 10^{-9}$. The unit of reluctance in the Giorgi system would be based on the reluctance of a cubic meter of free space, whereas in the C. G. S. S. system, it would be based on the cu. cm. Except in regard to the fundamental units of length and mass, there seems to be little to choose between the two systems. On the one hand, the Giorgi system has the meter and kilogram, while the C. G. S. S. has the cm. and the gram-seven or ten metric tons. The meter is preferable to the centimeter as a fundamental unit of length, on account of having no *centi* prefix to start with; but of course electrical workers are very familiar with the cm. by long habit. Again, the *kilogram* although encumbered with the prefix *kilo,* is preferable as a fundamental unit to the *gram-seven.* In fact Dellinger[22] has considered that in view of this awkward magnitude of unit mass, the C. G. S. S. system was not adapted for use in general scientific measures outside of electromagnetics. The unit of density in the C. G. S. S. system is of course 10^7 g. per cu. cm., or ten million times that of water. A book,* pamphlets, [20,24, 26, 28] and papers have appeared, written in the C. G. S. S. system; so that the C. G. S. S. system may claim to be a competitor of the M. K. S. Ω system, as a unified, rationalized, and comprehensive absolute system, embracing all the "practical" units, and quite independent of the original parent C. G. S. tree. The choice between them, if one or other should finally be adopted, probably lies as much with non-electrical as with electrical workers.

Moreover, in addition to the M. K. S. Ω and C. G. S. S., there are two other possible absolute systems embracing the "practical" units; namely, one based on the decimeter and 10^5 grams, the other based on the dekameter and dekagram. It was pointed out by Ascoli,* that since the kinetic energy of a mass M moving with a velocity V, is $M\ V^2/2$ units of dynamical energy, and since we desire to retain the fundamental time unit constant, in all these systems, as the mean solar second, then because the joule as a "practical" unit of dynamical energy has to be conserved at the magnitude of 10^7 ergs, we may vary the unit magnitudes of M and L so long as the product $M\ L^2$, expressed in C. G. S. values, remains constant at 10^7. Hence if M, the mass unit, is 10^m grams, and L, the length unit, is 10^n cm., we must keep $m + 2\ n = 7$. Restricting ourselves to positive values of m and n, *i. e.,* keeping within the range 1 cm. to 1000 cm. in L and from 1 g. to 10^7 g. in M, we have the C. G. S. S. Dellinger-Bennett system for $n = 0$ and $m = 7$; while we have the M. K. S. Ω Giorgi system for $n = 2$ and $m = 3$, the meter and the kilogram; but two other possible combinations are $n = 1$, $m = 5$ and $n = 3$, $m = 1$. Incidentally, it may be noticed that if we make $n = 9$ and $m = -11$, we obtain the Q. E. S. or quadrant-eleventh-gram-second system of Maxwell.

A comparative table of C. G. S., Giorgi, and C. G. S. S. units is appended (Table II).

Contribution to the Discussion on Future Desirable Action of the I. E. C. In what has preceded, the writer has endeavored to present the historical background of the magnetic-unit situation without prejudice or partiality. The following remarks are, however, submitted from the standpoint of an advocate for a particular course of I. E. C. action in the matter, as a contribution to the general discussion.

First. Before any definite new action is taken, it is desirable to standardize internationally the definitions of magnetomotive force, magnetizing force, magnetic reluctance, magnetic flux, and especially permeability, so as to reach international agreement upon the dimensions of permeability.

In the Giorgi and C. G. S. S. systems, it appears that permeability is a product $\mu_0\ \mu$ of space permeability μ_0, or the permeability of a vacuum, and specific permeability μ, that of the magnetic circuit with respect to free space. Space permeability μ_0 will probably be generally admitted to have dimensions; while specific permeability μ is probably a mere numeric. Perhaps on such a basis, definitions may be drawn which will secure universal endorsement.[30]

Second. It seems desirable to revise internationally the assignment of names to a few magnetic units in the C. G. S. system. With the dimensions of permeability established, the question whether the *gauss* can apply to both H and B can be decided. Concerning the *maxwell,* as adopted at Paris in 1900, there has been neither dispute nor confusion. It is true that in some countries the *maxwell* has been little used, but it has been considerably used in other countries. Its proper use should not be interdicted. Whatever new action may be taken in the near future, we must expect to see C. G. S. magnetic units continue to be used for a long time.

Third. It would be desirable to accept part of the Italian delegation's proposal at Bellagio in 1927, and adopt a "practical" unit of magnetic flux at the value of 10^8 C. G. S. units; but not to call it the *maxwell* which name has been reserved for the C. G. S. unit since 1900. Giorgi suggested the name *weber* for this "practical" unit, in his papers of 1901-1904, and Bennett also suggested the name *weber* for the same C. G. S. S. "practical" unit in 1917. The name would have to be agreed upon internationally. For the present, we may call this unit the *volt-second.*

It does not seem likely that the adoption of the *volt-second,* with its appropriate new name, would seriously conflict with the *maxwell* in practical use. Their magnitudes are so remote. The advantage would be that the path would be opened for electrical workers to employ either the Giorgi or the C. G. S. S. comprehensive practical system, without reference to the C. G. S.

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*Karapetoff's "The Magnetic Circuit" N. Y. 1911, although referring in its preface to the Giorgi system, appears to be written in the C. G. S. S. system and may perhaps be the first publication in that system.

*Ascoli M., Bibliography 16, p. 134.

units, if they desire to do so. The ampere-turn as a rationalized unit of m. m. f. in both these systems, already exists and probably needs no other name. The difficulty comes when an attempt is made to adopt the corresponding "practical" unit of flux density, whether with or without a special name. If the unit of flux density be chosen as the volt-second per square meter, it fits into the Giorgi M. K. S. Ω system, and virtually fixes upon that system exclusively. If, on the other hand, the unit of flux density is chosen as the volt-second per sq. cm., it fits in with the Karapetoff-Dellinger-Bennett "International" C. G. S. S. system, and logically would fix upon that system. It would seem to be sufficient at this time, to leave the path open for the use of a comprehensive system, without fixing upon the system, or on its series of units. If, however, a choice should be made at once, then in the opinion of the writer, the meter and kilogram are to be preferred over the centimeter and gram-seven.

If a comprehensive system such as the M. K. S. Ω system developed and found favor with electrical workers, it would greatly simplify electromagnetic theory, for teachers, students, readers, and writers, since we should all be able to abandon the existing duality of the C. G. S. and the derived "practical" units. It might well happen that the new comprehensive system would carry all before it, and relegate the C. G. S. system to history and the shelves, so far as electrical workers are concerned. In that case, the comprehensive system, being also adapted for use in mechanics, chemistry, optics, and all other branches of science, might readily invade those branches, and ultimately replace the C. G. S. units they use with M. K. S. Ω units. The numerical changes thus effected in tables and formulas, would be decimal changes, or a shift of the decimal point, in nearly all cases. Such a change, if it occurred, would probably come very slowly, and take many years to effect. Nevertheless, when it is remembered that all science is so closely interlaced through its units, that what happens to the units in one branch may readily react upon the unitology of other branches, it seems desirable not to take final steps of change in any region, without giving ample notice to the workers in others. All are interested, in all countries.

TABLE I

SUMMARY IN BRIEF TABULAR FORM OF PROPOSALS MADE BY THE VARIOUS MEMBERS OF THE I. E. C. INTERNATIONAL SUBCOMMITTEE ON MAGNETIC UNITS

	Quantity		1900 Paris Congress	British Delegate	French Delegate	German Delegate	Holland Delegate	Italian Delegate	U. S. A. Delegate
C. G. S. System (Magnetic)	Field Intensity	H	gauss	abolish all names and start afresh	mascart	weber	gauss	gauss	
	Magnetomotive Force	$\mathfrak{F}$							
	Flux	ϕ	maxwell			maxwell			maxwell
	Flux Density	B				gauss	maxwell/cm.^{2}		gauss
	Reluctance	$\mathfrak{R}$							
"Practical" System	Field Intensity	H				(Amp./cm.) no special names proposed	gilbert ←A. I. E. E. unrationalized system as proposed in 1893		
	Magnetomotive Force	$\mathfrak{F}$							
	Flux	ϕ			no name	(volt-second)	weber or maxwell	maxwell	
	Flux Density	B				(volt-sec./cm.^{2})			
	Reluctance	$\mathfrak{R}$					oersted		

At present, no comprehensive system of units can make headway in the magnetic circuit, so long as the magnetic-circuit units are by international agreement wholly in the C. G. S. system. If a new unit of flux is created, with a name, in the "practical" series, that obstacle would be removed. If the comprehensive system utilizes this, and develops useful service to the world in the next few years, then subsequent I. E. C. action can adopt the comprehensive system, and set the C. G. S. system aside. If, however, for any reason the comprehensive system fails to find favor, the volt-second unit will probably not come into use, and might later be withdrawn. The C. G. S. system might then continue.

Remarks upon the Tables and their Contents. Table I gives a brief summary of the proposals made by the different members of the I. E. C. International Subcommittee on Magnetic Units, in their exchange of correspondence. It is evident that their opinions differed widely as to what recommendation should be reported to the I. E. C. It is, however, needless to say that it is impossible to do adequate justice to a series of fairly long and carefully considered written documents in a simple skeleton chart of this kind. If it had been possible to hold meetings and discussions, it might have been possible to come to a closer range of proposals. The subcommittee report, however, as appended, met with general agreement.

Table II offers a comparison, in parallel columns, of corresponding units in the fundamental C. G. S. magnetic system and three "practical" systems, *i. e.*, the Q. E. S. system of Maxwell, the Giorgi system, and the more recent "International" system of Dellinger, Bennett, and Karapetoff. The C. G. S. electrostatic system has been omitted to save space.

All three "practical" systems are represented as being

TABLE II

TABLE OF FUNDAMENTAL C. G. S. UNITS AND OF THREE DERIVED "PRACTICAL" SYSTEMS

			Fundamental	← Practical					→
No. I	Quantity II	Symbol III	C. G. S. IV	Q. E. S Maxwell, 1881 V	In C.G.S.U. VI	M. K. S. Ω Giorgi, 1901 VII	In C.G.S.U. VIII	C. G. S. S. Dellinger-Bennett IX	In C.G.S.U. X
	Mechanic								
1	Length	L	*cm.*	quadrant	10^9	*meter*	10^2	*cm.*	1
2	Mass	M	*g.*	eleventh-g.	10^{-11}	*kilogram*	10^3	g.-seven	10^7
3	Time	T	*second*	second	1	*second*	1	*second*	1
4	Area	S	*cm.*^{2}	sq. quad.	10^{18}	*m.*^{2}	10^4	*cm.*^{2}	1
5	Volume	V	*cm.*^{3}	cubic quad.	10^{27}	*m.*^{3}	10^6	*cm.*^{3}	1
6	Density	d or ρ	*gm./cm.*^{3}	eleventh g/quad.^{3}	10^{-38}	*kg/m.*^{3}	10^{-3}	*g*-seven/*cm.*^{3}	10^7
7	Velocity	v	*cm./sec.*	quad./sec.	10^9	*m/sec.*	10^2	*cm/sec.*	1
8	Acceleration	a	*cm./sec.*^{2}	quad./sec.^{2}	10^9	*m/sec.*^{2}	10^2	*cm/sec.*^{2}	1
9	Force	F	*dyne*	centidyne	10^{-2}	dyne-five	10^5	dyne-seven	10^7
10	Pressure	p	*dyne/sq. cm.*	cntdyne/sq. quad.	10^{-20}	dyne-5/sq. *m.*	10	dyne-7/sq. cm.	10^7
11	Torque	Q	*dyne* ⊥ *cm.*	cntdyne ⊥ quad.	10^7	dyne-5 ⊥ *m.*	10^7	dyne-7 ⊥ cm.	10^7
12	Moment of Inertia	J	*g.-cm.*^{2}	11th-g-quad.^{2}	10^7	*kg-m.*^{2}	10^7	*g-cm.*^{2}	1
	Energetic								
13	Work	W	*erg*	*joule*	10^7	*joule*	10^7	*joule*	10^7
14	Power	P	*erg/sec.*	*watt*	10^7	*watt*	10^7	*watt*	10^7
	Thermal								
15	Heat	H	*g-calorie*	11th-g-cal.	10^{-11}	*kg-calorie*	10^3	*g-calorie*	1
16	Temperature	t or T	*Deg. C. or Abs.*	*Deg. C. or Abs*	1	*Deg. C. or Abs.*	1	*Deg. C. or Abs.*	1
	Luminous								
17	Flux	F	*lumen*	*lumen*	1	*lumen*	1	*lumen*	1
18	Illumination	E	*phot*	lumen/quad.^{2}	10^{-18}	*lux*, lumen/m.^{2}	10^{-4}	*phot*	1
19	Intensity	I	*int. candle*	*int. candle*	1	*int. candle*	1	*int. candle*	1
20	Brightness	B	candles/cm.^{2}	candles/quad.^{2}	10^{-18}	candles/m.^{2}	10^{-4}	candles/cm.^{2}	1
21	Focal Power		cm.$^{-1}$	quad.$^{-1}$	10^{-9}	*diopter*	10^{-2}	cm.$^{-1}$	1
	Electric								
22	Electromotive Force	E	abvolt	*volt*	10^8	*volt*	10^8	*volt*	10^8
23	El: field intensity	e or V	abvolt/cm.	volt/quad.	10^{-1}	volt/m.	10^{-2}	volt/cm.	10^8
24	Resistance	R	abohm	*ohm*	10^9	*ohm*	10^9	*ohm*	10^9
25	Resistivity	ρ	abohm-cm.	ohm-quad.	10^{18}	ohm-m.	10^{11}	ohm-cm.	10^9
26	Current	I	abampere	*ampere*	10^{-1}	*ampere*	10^{-1}	*ampere*	10^{-1}
27	Current density	i	abamp/cm.^{2}	amp./sq. quad.	10^{-19}	amp./sq. m.	10^{-5}	amp./sq. cm.	10^{-1}
28	Conductance	G	abmho	mho	10^{-9}	mho	10^{-9}	mho	10^{-9}
29	Conductivity	γ	abmho/cm.	mho/quad.	10^{-18}	mho/m.	10^{-11}	mho/cm.	10^{-9}
30	El: Quantity	Q	abcoulomb	*coulomb*	10^{-1}	*coulomb*	10^{-1}	*coulomb*	10^{-1}
31	El: Displacement	D	abcoulomb/cm.^{2}	coulomb/quad.^{2}	10^{-19}	coulomb/m.^{2}	10^{-5}	coulomb/cm.^{2}	10^{-1}
32	Capacitance	C	abfarad	*farad*	10^{-9}	*farad*	10^{-9}	*farad*	10^{-9}
33	Permittivity	κ_o	(abfarad/cm.) = 1	farad/quad. =	$\frac{1}{4\pi v^2}$	farad/m. =	$\frac{10^7}{4\pi v^2}$	farad/cm. =	$\frac{10^9}{4\pi v^2}$
34	Frequency	f	cycle/sec.	cycle/sec.	1	cycle/sec.	1	cycle/sec.	1
	Magnetic								
35	Magnetomotive Force	$\mathfrak{F}$	gilbert	*amp.-turn*	$\frac{4\pi}{10}$	*amp.-turn*	$\frac{4\pi}{10}$	*amp.-turn*	$\frac{4\pi}{10}$
36	Mag: field intensity	H	gilbert/cm.	amp-turn/quad.	$4\pi/10^{10}$	amp-turn/m.	$4\pi/10^3$	amp-turn/cm.	$4\pi/10$
37	Space Permeability	μ_o	abhenry/cm.	henry/quad.	$1/4\pi$	henry/m.	$10^7/4\pi$	henry/cm.	$10^9/4\pi$
38	Magnetic Flux	Φ	*maxwell*	volt-second	10^8	volt-second	10^8	volt-second	10^8
39	Mag. Flux Density	B	maxwell/cm.^{2}	volt-sec./quad.^{2}	10^{-10}	volt-sec./m.^{2}	10^4	volt-sec./cm.^{2}	10^8
40	Permeance	$\mathcal{P}$	(oersted)$^{-1}$	henry	$10^9/4\pi$	henry	$10^9/4\pi$	henry	$10^9/4\pi$
41	Reluctance	$\mathcal{R}$	oersted	yrneh	$4\pi/10^9$	yrneh	$4\pi/10^9$	yrneh	$4\pi/10^9$
42	Inductance	L	abhenry	*henry*	10^9	*henry*	10^9	*henry*	10^9
43	Magnetization	I		volt-sec./quad.^{2}	$10^{-10}/4\pi$	volt-sec./m.^{2}	$10^4/4\pi$	volt-sec./cm.^{2}	$10^8/4\pi$
44	Magnetic Pole	m	maxwell/4 π	volt-sec.	$10^8 \times 4\pi$	volt-sec.	$10^8 \times 4\pi$	volt-sec.	$10^8 \times 4\pi$
	Chemical								
45	Electrochemical Equivalent		*g*/abcoulomb	11th-*g*/coulomb	10^{10}	Kg/coulomb	10^{-4}	g-seven/coulomb	10^{-8}

rationalized by the removal of the 4π factor from magnetomotive force and inserting it into reluctance, so as to keep the unit of flux correct. This method of rationalization did not become available until about 1893, and did not appear in the original scheme of Maxwell; but the comparative relations of the three systems are best presented by applying the same rationalization to all alike. Likewise, all three are presented as unified systems through the Giorgi device of adopting suitable values for the space coefficients κ_0 and μ_0.

The units appearing in italics in the table have names that have been internationally adopted, or may claim to have met with international recognition. The names in Roman type, on the contrary, have not been adopted internationally, or are descriptive only.

The numerical values appearing in Columns VI, VIII, and X indicate the number of C. G. S. units contained in the practical unit considered, except in the case of κ_0, No. 33, which gives the values of that coefficient in the three different "practical" systems.

It will be observed that all three "practical" systems have the following electric, magnetic, and energetic units in common: the *joule, watt, volt, ohm, ampere, coulomb, henry, and ampere-turn,* if the last named may be properly regarded as an international unit. In many of the other units, however, characteristic differences of magnitude appear. Since it is unlikely that anyone will recommend the Q. E. S. system for practical use throughout, owing to the very awkward values of its length and mass units, we may confine attention to the

TABLE III

NUMERICAL VALUES OF SPACE PERMITTIVITY κ_0 AND OF SPACE PERMEABILITY μ_0 IN THE Q. E. S., M. K. S. Ω, AND C. G. S. S. "PRACTICAL" SYSTEMS

*	$n = 9$ $m = -11$ Q. E. S. Maxwell, 1881	$n = 2$ $m = 3$ M. K. S. Ω Giorgi, 1901	$n = 0$ $m = 7$ C. G. S. S. or "International" Dellinger-Bennett
Space Permittivity κ_0	$\frac{1}{4 \pi v^2}$ with $v = 29.98$ 8.85×10^{-5}	$\frac{10^7}{4 \pi v^2}$ with $v = 2.998 \times 10^8$ 8.85×10^{-12}	$\frac{10^9}{4 \pi v^2}$ with $v = 2.998 \times 10^{10}$ 8.85×10^{-14}
Space Permeability μ_0	4π 12.566	$4 \pi \times 10^{-7}$ 1.2566×10^{-6}	$4 \pi \times 10^{-9}$ 1.2566×10^{-8}

*All of the "practical" units thus far (1929) adopted internationally conform to any one of a theoretically infinite series of $L\,M$ ($T = 1$ sec.) systems, in which $L = 10^n$ cm., and $M = 10^m$ g.; such that $2n + m = 7$. The adoption of the "volt-second" practical unit of flux does not affect this generalization. It seems, however, that the international adoption of a corresponding "practical" unit for either H or B will logically select the particular "practical" system, as M. K. S. Ω or C. G. S. S. for the entire series of Table II, according as the meter or centimeter is taken for length and cross-section.

M. K. S. Ω and C. G. S. S. systems, in Columns VII and IX.

Among the electric and magnetic units, 22 to 44, inclusive, there is not much to choose between the two systems of Columns VII and IX. Indeed, electrical workers may prefer the centimetric values of Col. IX to the metric values of Col. VII, because by long familiarity with the fundamental C. G. S. system, the centimetric operations are familiar. There is likewise little advantage on either side in the thermal and luminous lists. The principal differences are found in the mechanic series, particularly in Mass and in Density, Nos. 2 and 6. Here the M. K. S. Ω system has distinct advantages. The unit mass of ten metric tons, or 10^7 grams, is a serious handicap to the C. G. S. S. system. Consequently, if the "practical" system were restricted entirely to the use of electrical workers, with problems involving mass seldom presenting themselves, either of the two systems might be adopted, to the ultimate exclusion of the fundamental C. G. S. system. There might then be two remaining systems in scientific work, one the "practical" system for electrical workers, and the other the C. G. S. system for non-electrical workers, or all other scientists. It seems more likely that the M. K. S. Ω system would gain the favor of all the non-electrical workers, than the C. G. S. S. system with its large unitary mass. In other words, the greater hope for a final single scientific system of units in the distant future, seems to lie on the side of the M. K. S. Ω system. At all events, this question deserves careful consideration before a "practical" unit of magnetic intensity H, or flux density B is internationally adopted.

Table III presents the numerical values of κ_0 and μ_0 in the three "practical" systems.

Appendix

Report of International Subcommittee on Magnetic Units

To Dr. C. O. Mailloux, President, I. E. C. Committee on Nomenclature—

The Subcommittee on Magnetic Units has considered the questions proposed to it at the Bellagio meeting of 1927, as follows:

(1) In view of certain differences of usage of the magnetic unit *gauss* in different countries, can we recommend any action directed towards unification of usage in the future?

(2) In view of the communication of the Comité Electrotechnique Italien [1 (Italy) 27], presented at Bellagio, what recommendations are we able to make?

The subcommittee has discussed these questions by correspondence only.

The subcommittee recommends (1) that steps should be taken towards unification of the definitions of the magnetic units and of their names. The subcommittee has not yet been able to arrive at an agreement on the specific steps that should be thus taken. It recommends that the Committee on Nomenclature of the I. E. C. should place this important question on the agenda for its next international meeting, and should seek to elicit discussion upon the question in the various national committees, in advance of the meeting, so as to pave the way for well considered international action.

(2) The subcommittee is unable to recommend the suggestion in the proposal of the Italian Committee [1 (Italy) 27], to change the name *maxwell* from the C. G. S. U. of magnetic flux ϕ to the "practical" or volt-ampere-ohm unit of magnetic flux ϕ, in view of the dislocation in understanding of electromagnetic literature that might thereby be brought about.

Signed
W. H. Eccles
P. Janet
A. E. Kennelly, Chairman,
L. Lombardi
Mitkievitch
K. Strecker
G. J. Van de Well

24th September, 1929.

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Discussion

W. J. Shackelton: The American Society for Testing Materials has a standing committee whose responsibility it is to formulate specifications describing approved methods of testing materials for such magnetic properties as are of mutual concern to the two major classes of its members, that is to say, to suppliers and consumers. The use of such uniform methods in the inspection of material supplied under a specification of magnetic quality is a matter of practical necessity. In its work this committee, of course, has had to use magnetic units and for many years has recognized that the situation with respect to definitions and names has not been a very happy one. This is well brought out by the historical outline contained in Dr. Kennelly's paper.

Some three years ago the committee, in view of its special needs, undertook a compilation of a glossary of the commonly used magnetic terms with their definitions. The result of this undertaking has been:

1. The publication in 1927 of a preliminary list of terms and definitions on which criticism and comment were requested.
2. The publication in 1928 of a revised and enlarged list, also as a matter of information and comment.
3. The publication in 1929 of the list again slightly revised as a tentative standard of the society.

Quite recently this list has been submitted to the newly formed Sectional Committee on Electrical Definitions of which Dr. Kennelly is the chairman.

The A. S. T. M. Committee throughout its study of the subject has approached the problem from the standpoint of selecting wherever possible that which is most nearly representative of good general practise now prevailing. In the matter of units it found that there was a sufficiently well defined usage to indicate which should be formally recognized. These, of course, do not include the "ampere-turn" or the "volt-second," although the committee recognizes that these units have some distinctive merits. On the question of definitions, however, no substantial agreement could be found in the literature; a situation which is also pointed out by Dr. Kennelly. It was necessary, therefore, for the committee to strike out to a certain extent for itself, and after a considerable amount of discussion among those evidencing an interest in the subject, the definitions as published were set up.

In the light of our experience in this connection I can definitely second Dr. Kennelly's conclusion as to the desirability, in fact I should say the necessity, of obtaining agreement on definitions, at least of magnetomotive force and magnetic flux, before much lasting progress can be made toward a uniform system of units and names. Comparing the tentative definitions of the A. S. T. M. with the specific proposals made by Dr. Kennelly, I do not find any conflict in substance. It may be that all will not wish to subscribe to these definitions exactly as now worded, but it would seem that they might well serve as a starting point for the general agreement which it is so desirable to have.

M. G. Lloyd: Dr. Kennelly has pointed out the different views with respect to the unit in which H should be measured, and it seems to me those different views are perhaps rooted in a difference in the idea which is represented by this symbol H. I think there are two distinct things represented by that symbol in different cases, and because they have the same numerical value they have been confused.

In Table II, on the seventh page of the paper, Item No. 36 is listed as magnetic field intensity and the symbol H is given and the unit is given as gilbert/cm. If H represents the resulting flux in air it seems to me it has the same dimensions as B and should be measured in the same unit. They are both flux densities. One is in air, or in a vacuum perhaps, which has a permeability *one* which we have assigned to it. The other is in a medium of different permeability, but in each case there is a resultant flux, which has a flux density. Because the permeability happens to be unity in one does not make it disappear in terms of dimensions. The magnetic flux density in air has the same dimensions and can be measured in the same unit as magnetic flux density in iron.

However, gilberts per cm. is a different quantity, if permeability has dimensions. Different people have called them both H. Perhaps the same person has called them both H. Possibly we need to distinguish between our gilbert/cm., the magnetomotive force per unit length, and the resultant flux from it in the medium which has permeability unity. I think perhaps that difficulty is at the root of the confusion.

I want to endorse the recommendation of the committee, that if a different unit is wanted for the magnetic flux we should not attempt to carry the term now well known over to a new magnitude, but we should find a new name for that new unit, and let the maxwell either be continued as it is at present, or forgotten if we all come to another unit.

C. L. Dawes: During the last fifteen years immense progress has been made in the matter of international standardization, and it comes, I believe, as a distinct surprise to a number of us to learn that no international agreement has as yet been reached in the definitions of such very fundamental scientific quantities as magnetic units. It appears that the first attempts to define electrical quantities were made as far back as 1861, and the first specific attempts to define magnetic quantities were made in 1889. Although various conferences for standardizing units have been held over a period of forty years, such fundamental quantities as magnetic units have not as yet been standardized internationally.

The purpose of this paper, as is stated by Dr. Kennelly, is to obtain individual opinions from members of the Institute on three specific recommendations which he proposes. With any substantial support by Institute members and other scientific organizations, he may be able to effect an agreement with the other members of the International Subcommittee, on certain definitions, and hence make a beginning, at least, toward international accord on magnetic definitions. I personally am in hearty agreement with all three recommendations.

Referring to the first suggestion, dimension, I believe, should be assigned to the quantity μ. The fact that the dimensions of a quantity cannot be expressed immediately in the fundamental units of length, mass, and time does not mean necessarily that the quantity has no dimension, but rather that our present knowledge is too inadequate to assign dimensions. For example, many believe that temperature has dimensions, but so far, dimensions in units of length, mass, and time cannot be assigned to it. We also know that $\sqrt{\mu \kappa}$, where κ is dielectric constant, is equal to the reciprocal of a velocity which has dimensions. This implies that μ is dimensional. Hence, in view of the present limits of our knowledge, I believe that it would be safer to consider μ a dimensional quantity, and if at some future time it is found to have no dimensions, the quantity could then be considered a numeric, without any harm having been done. If then, μ is considered as a dimensional quantity, B and H cannot have the same dimensions. I agree absolutely with Dr. Lloyd that most of the confusion in considering $B = H$ comes from the fact that the gilberts per cm. are equal numerically to the flux density in air. If the fact that these quantities are only equal numerically, were clarified, I believe that most of this confusion would disappear. This may be done very readily by adopting the suggestion of Dr. Kennelly in assigning to free space a permeability μ_o, a dimensional quantity, and considering μ, relative permeability, a mere numeric. In the second suggestion Dr. Kennelly asks merely that the name *maxwell* be applied to the unit of flux and he feels that the other members of the committee would concur in this. In view of accepted usage in this country, I do not believe that any objection can be raised to this. Moreover, I believe that the further recommendation should be made that the unit of flux density be called the *gauss*.

The third suggestion refers to a proposal by the Italian delegate that the name volt-second ($= 10^8$ c. g. s. lines) be applied to the practical unit of magnetic flux.

In international conferences of any character, it is usually necessary that everyone make some concessions. We are asking that names of units which have acquired usage in this country be applied to some of the magnetic units, and delegates from other countries are asking consideration of their recommendations. Hence, I not only see no harm in accepting this terminology proposed by the Italian delegate, but feel that this country should welcome such a practical unit. Moreover, if agreeable to the committee I should be in favor of calling this unit the *weber*. This unit is advantageous in that it eliminates the factor 10^8 which enters so many engineering calculations.

The opinion has been expressed that if such a practical unit were adopted, the fundamental c. g. s. system would thereby be eliminated. From its very nature, it is almost impossible to eliminate any fundamental system of units. As Dr. Kennelly has said, if this ever should happen, it would be at a remote period and I do not believe at this time that we need be concerned with it.

This paper, by outlining so concisely the history of the development of magnetic units and by focusing attention on their present status, should give a much needed impetus to a final standardization of magnetic units internationally.

F. W. Gay: The use of units to be employed in any given instance has often been considered as merely the matter of placing a decimal or the question of whether the multiplier between units should be 10, 12, or some other number.

There are far greater basic factors affecting the choice of units. These factors may perhaps be arranged in two great groups.

1. *Psychological.* It is undoubtedly a fact that for any particular class of work there is a particular dimension for the basic unit which will give the best result in aiding the engineer or practical man in his calculating work. A smaller unit would give too large a figure for ready mental calculation while a larger unit would often drive him to the use of decimals.

2. *Practical.* It is quite possible that an entirely different unit may be desirable from the standpoint of the workman who performs actual mechanical work, since he does little figuring and is only interested in the meeting of dimensions within the required tolerance.

As an example of the first instance it might be stated that the centimeter has not proved the equal of the inch in ordinary magnetic calculations involving design work. The unit appears to be so small that the figures which must be mentally multiplied and divided are too large for the mental arithmetic of the ordinary engineer. The inch appears to be a much better unit.

As an example of the second sort it might be wise to consider the industrial prosperity of the United States. The unit of measurement which has given the United States its present industrial position is the mil, the one thousandth part of an inch. The fact that years ago the American workmen set up the mil as a tolerance standard to work to and the industrial manager has determined to obtain a high output accurate to one mil, has resulted in the interchangeability of parts and all that such interchangeability means.

If we look abroad we will find that the metric system either has a logical tolerance value approximately four times as great or only four-tenths as great as our mil. It is well known that the Swiss workman works to the smaller tolerance unit, that is, he works to a much greater accuracy than that used in the United States. His workmanship is very fine but his output is so small that it will not compete with the output of the United States in the same line of work. In some other foreign countries the tolerance standard is four times as great as in the United States, that is, it is approximately four mils and this degree of tolerance is so liberal that interchangeability of parts is not possible. In consequence it is necessary to fit articles by hand and costs are correspondingly increased.

Of course it can be argued that even tolerances are not necessary. The fact nevertheless remains that they are convenient tolerances, which are automatically set up as standards once a system is chosen.

It is altogether possible that the standards of workmanship obtainable in the United States will be pushed up so high that a tolerance of four-tenths as great as at present can be used without a great increase in cost per unit. However, it would seem that we are a long way from such a desirable condition and hence we are a long way from the time when the metric system should be adopted as our standard.

In conclusion we should like to point out that it is more important for foreign countries to change to a commercially sound and practical system of measurements than for the United States to change. It is undoubtedly desirable that an international system should be adopted; but the possibilities for good or evil both upon the engineers who must think in the system and upon the morale of the workman who must work in the system are so great that the prosperity of a nation may be seriously affected by an unwise choice.

When the electromagnetic fundamental units have once been standardized it might be wise to carry forward time studies looking towards picking a practical system of magnetic units based perhaps on the ampere turn and the reluctance of a cubic inch of air.

V. Karapetoff: I most heartily second Dr. Kennelly's wish that a new unit of flux be introduced to agree with the practical ampere-ohm system of units in general use. The question whether it should be called the weber or the volt-second is not so important, although I have used the term weber in my "Magnetic Circuit" (McGraw-Hill) since 1911.

I should like to go a step further and ask that the ampere-turn be recognized as the practical unit of magnetomotive force, because this would automatically fix the magnitude and the physical dimension of permeability. The fundamental equation of the magnetic circuit, corresponding to Ohm's law in the electric circuit, is

$$\Phi = M\,P \tag{1}$$

where Φ is the flux, M the magnetomotive force, and P the permeance of the circuit. For a circuit of uniform cross-section

$$P = \mu\,A/l \tag{2}$$

where l is the length, A the cross-section of the circuit, and μ its absolute permeability. Substituting this value of P from Equation (2) in Equation (1) and dividing both sides by A, gives

$$\Phi/A = \mu\,M/l \tag{3}$$

or simply

$$\mu = B/H \tag{4}$$

Thus, the dimension of μ is the ratio of those of B and H. If B is expressed in volt-seconds per sq. cm., its dimension is $R\,I\,T\,L^{-2}$, where I is the dimension of the current and R that of the electrical resistance; L and T are length and time respectively, and with their units we are not here concerned. The dimension of H in ampere-turns is $I\,L^{-1}$. Therefore, the dimension of μ in the practical ampere-ohm system is

$$\mu = R\,T\,L^{-1} \tag{5}$$

and not a numeric ("The Magnetic Circuit," p. 263). We may now put

$$\mu = \mu_r\,\mu_v \tag{6}$$

where μ_r is the relative permeability of a material and μ_v is the absolute permeability of vacuum. In this manner, the usual values of μ can be retained, only they become values of relative permeabilities whose dimension is a numeric.

Professor McKeehan's argument that the physical dimensions of B and H are the same, because of the formula

$$B = H + 4\,\pi\,J \tag{7}$$

is not convincing. This formula was originally deduced on the supposition that $\mu = 1$. In any other system of units the formula would read

$$B = \mu_v\,H + k\,J \tag{8}$$

I was surprised to learn from Dr. Kennelly's computations that the unit of mass in the system of magnetic units which I have been advocating and using for over twenty years is ten metric tons. To me this simply shows that in framing a convenient system of electric or magnetic units, a convenient unit of mass or length is not essential, because it is so seldom used in conjunction with electric units. It is better to have a convenient system of electric and magnetic units, and to use a decimal factor for the mass when it comes in on rare occasions, than to be forced to a system of inconvenient electric units for the sake of basing them on the centimeter and the gram.

No one deplores the existing multiplicity of magnetic units more than I do, yet looking back upon the progress of our art and of the adjacent branches of physics, it does not seem to me that the progress has been materially impeded by a lack of agreement about units, nor that serious misunderstandings have ever arisen in this respect. I am not trying to belittle the praiseworthy efforts of Dr. Kennelly and his colleagues on the I. E. C., but partly to re-assure outsiders that no critical (or near-critical) situation exists, and partly as a possible explanation of the exceedingly slow progress made by the Commission with respect to magnetic units. Had an urgent need existed for an agreement in regard to magnetic units, I am sure that speedier progress would have been made.

The real situation is that each group of workers using magnetic quantities employs a few units which it finds convenient, and is not troubled by the fact that the corresponding unit of length in that system is one-quarter of the terrestrial meridian, nor that the density of water comes out to be quite an unusual figure. Much work is done in algebraic symbols, without even specifying the system of units used, and the results are usually correct in any consistent system of units.

It seems to me that the problem of units could be considerably simplified if we agreed beforehand to accept a unit with all its multiples and submultiples in powers of ten as belonging to the same system. For example, I see no particular reason for arguing whether the meter or the centimeter is preferable as the fundamental unit of length. In applications we have to use lengths from billionths of a millimeter to billions of kilometers, and yet no confusion arises because all these lengths are connected by simple factors equal to ten to some integral power. The same applies to the electric and magnetic units, and I see no valid reason for arguing whether the maxwell or the weber should be the fundamental unit of flux, as the two are connected by the factor equal to ten to the eighth power. No matter which one is decided upon, both will be used in applications, as well as kilo-maxwells, mega-maxwells, milli-webers, etc. I may be missing an essential factor from the point of view of the general philosophy of units, but from the standpoint of a user I am only mildly interested as to which of the two is decided upon. Of course, re-naming the bigger unit the maxwell is like re-christening the Washington Monument the Lincoln Monument, and the proposal will never be approved by a judicious and conservative body like the I. E. C.

When, however, it comes to units whose ratio is 3×10^n or $4\,\pi$, the situation is more troublesome, and I heartily support Dr. Kennelly's recommendation that the whole question of the theory of the magnetic circuit be first gone over, especially with regard to the physical dimensions of permeability, which is really the crux of the whole situation. The commission should state that B is to be measured in flux units per unit area and H in current-turns per unit length; this will at once give physical dimensions to the absolute permeability, and will permit differentiation between the absolute permeability of a medium and its relative permeability with respect to that of the air, as I showed above. The numerical value of the absolute permeability of the air (or vacuum) will still depend upon the system of units used.

Dr. Kennelly dismisses the c. g. s. electrostatic system from his considerations, and perhaps he is right in so far as its use in power engineering is concerned. However, it is also of importance to consider the needs of physicists. At least I, for one, have always considered that the functions of the government delegates to the I. E. C. included the needs of pure science as well. Moreover, the progress of the engineering art is essentially determined by advances in physics and chemistry, and it would be quite unfortunate if the final definitions and units adopted by the I. E. C. were not acceptable to research physicists. I hope that in the final draft of his paper Dr. Kennelly will include the history of the two other systems of units now largely used in books and articles on theoretical physics, namely the Gauss system[1] and the Heaviside-Lorentz system.[2] The first system permits the use of the electric quantities in an electrostatic system of units and the magnetic quantities in an electromagnetic system of units, in the same equation. The second system accomplishes the same purpose and also enables us to avoid an "eruption of 4 π's."

With the advent of the electronic theory and with a growing interest in electromagnetic waves in space, equations are used more and more in which electrostatic quantities, such as the charge on an ion and the electric field intensity, are combined with magnetic quantities, H and B. Therefore, in considering various systems of magnetic units it is essential, for the sake of future workers, to provide convenient relations with electrostatic quantities. The Gauss system is largely used by mathematical physicists; therefore, it is only a question of time when research engineers will also be using it in continuing and applying the work of physicists to their problems. As a rule, it does not do to anticipate a system of units ahead of its actual use, but here we undoubtedly are facing an inevitable invasion and it may be wise to be prepared for it.

I sincerely hope that the I. E. C. will adopt the policy of "decimal families of units," not insisting on a particular unit, but including all its multiples and sub-multiples in powers of ten as well. I also hope that the status of the absolute permeability and absolute permittivity will be settled at least in so far as recognizing them to be dimensional quantities and not pure numerics.

Edward Bennett: (communicated after adjournment) The status and the historical background, not only of the magnetic circuit units, but also of the proposals looking to an agreement as to the units of length, mass, and force in the system of international units are so clearly and fairly presented in Dr. Kennelly's paper that his plea for full discussion to pave the way for well considered international action should meet with the heartiest response. My discussion will be confined to the principles which should be applied in passing upon the merits of alternative proposals and to some comparisons in the light of these principles.

The aim in devising a *comprehensive system* of units is to *reduce mental effort* in the fields of thought which have been explored, in order that men may be free and fresh for adventures in virgin fields.

Importance of Unencumbered Formulas. In the devising of a system of units, the process is to adopt the smallest possible number of *arbitrarily defined independent* units to serve as the *fundamental* units of the system, and to define the units for all other physical quantities in the simplest and most comprehensible manner by means of these fundamental units. The scores of units which in this manner are defined by reference to the fundamental units are called the *derived* units of the system.

Now the unit for one physical quantity can be defined in terms of other physical quantities only by means of known relations between the quantities. Accordingly, the use of a *system* of derived units makes it possible to reduce mental effort, *provided* these known relations are properly utilized in defining the *derived* units. Proper utilization means that the derived units must be so defined that the important equations, or formulas, used in predicting the values of unknown quantities are not *encumbered* by needless or perplexing factors which are burdensome to the memory.

The difference between thinking in terms of the haphazard *set* of arbitrarily defined units of a primitive culture and in terms of a *system* of derived units is this: In the former case, every quantitative relation or formula is the expression of an empirically or experimentally determined relation, and consequently every formula is *encumbered* with a non-significant numerical factor. In the latter case, the experimentally determined relations are so used in defining the quantities and in defining the units as to leave many of the important formulas free of encumbering factors. In other words, those relations which are *matters of definition* are made to play a simplifying role. The difference between the two situations, thinking in terms of encumbered and unencumbered formulas, may seem slight. But it may be recalled that the Scriptures portray the prophet of old as smiting with his rod to cause the life-giving water to flow from the rock. Well, the prophet of today is the scientist, the formulas constitute his rod, and encumbered formulas make an ineffective rod,—at least for the minor prophets.

Any system of units falls short of the ideal to the extent to which it is encumbered by the presence of needless numerical factors in the formulas expressing the relations between quantities.

Importance of Units of Convenient Size. If the first desideratum in a system of units is that the formulas of the system shall be unencumbered, certainly the second is that the units for which the formulas have been written shall be of such a size as to lead to *convenient numerical expressions* for the quantities dealt with in the shop, the laboratory, and the market. It is particularly desirable that the *formula units* of the quantities dealt with in the every-day affairs of the common life of the community shall be of *convenient and readily comprehensible size.*

In this discussion, it is important to have a clear understanding of the term *formula unit.* In the formulas of a comprehensive and completely consistent system, any physical quantity such as length,—no matter in what formula it appears, whether in a formula for area, velocity, force, energy, temperature gradient, electric intensity, or what not,—*will always be expressed in the same unit*, say the meter. This unit is termed the *formula unit* of length in that system.

If past experience with systems of units teaches any lesson it is that the *formula units* of any comprehensive system cannot all be of convenient magnitude for the varied needs of the shop, laboratory, and market. For example, if the *formula unit* of length is the *meter*, men may be expected to measure cloth and town lots in meters but they can not be expected to dimension machine shop drawings or talk about the thickness of paper, or automobile speeds, or city to city distances in meters. As further illustrations:

The *joule* or the *watt-second* is not a convenient unit for use in billing energy and so energy is measured and billed in *kilowatt-hours.*

Transformers are rated and sold in terms of *kilowatts* or *kilovolt-amperes.*

Condensers are rated and sold in terms of *microfarads.*

Machine shop drawings are dimensioned in *millimeters.*

Insulation resistances are stated in megohms and metallic resistivities in *microhms.*

And so it happens that no matter how comprehensible the formula units may be, a great number of shop, laboratory, and market (s. l. a. m.) units will come into use. There is not the slightest possibility of preventing these s. l. a. m. units from springing into use. Nor should there be any desire to suppress or to discourage their use, *provided,*

1. That these s. l. a. m. units are decimal multiples or sub-multiples of the formula units,

2. That the ratio of each s. l. a. m. unit to its corresponding formula unit is unambiguously shown by a consistent use of the system of decimal prefixes in forming the name of the s. l. a. m. unit from that of the formula unit.

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[1] Geiger und Scheel, *Handbuch der Physik*, Vol. 16, 1927, pp. 4 to 13.

[2] Leigh Page, "Introduction to Theoretical Physics." 1928, pp. 424 aud 432.

The lesson from these illustrations is that the use of decimal multiples and submultiples of those units which are of inconvenient size for the transactions of commerce is readily accepted, or is an easy thing to bring about. The appearance of the prefix micro, milli, or kilo in the name of a s. l. a. m. unit does not militate against the use of the unit by the uninformed man in the shop, laboratory, and market. To the uninformed man, the prefix "kilo" is a part of the name, and the two syllables mean no more to him than do two syllables in any polysyllabic name. But to the informed man, the prefix "kilo" should not be a part of the name of the unit. The use of the decimal prefixes should be so consistent, that the informed man may always treat the prefix as a part of the *numeric*, and may always regard the name following the prefix as the name of the *formula unit* of the system.

Since there are gaps in the list of decimal prefixes, I venture to propose that these gaps be filled by the prefixes which are printed in italics in Table I.

TABLE I
DECIMAL PREFIXES FOR FORMING THE NAMES OF THE S. L. A. M. UNITS FROM THE FORMULA UNITS

Subdivisions			Multiples		
Prefix	Symbol	Multiple	Prefix	Symbol	Multiple
deci	dc	10^{-1}	deka	dk	10
centi	c	10^{-2}	hekto	hk	10^2
milli	m or ml	10^{-3}	kilo	k	10^3
*demil**	dml	10^{-4}	myria	my	10^4
cemil†	cml	10^{-5}	*penta*¶	p	10^5
micro	μ or mc	10^{-6}	mega	mg	10^6
billi‡	bl	10^{-9}	*kilomega*	kmg	10^9
Micro micro§	μ μ or mcmc	10^{-12}	*mega mega*	mgmg	10^{12}

*from decimilli.

†from centimilli.

‡billi from billionth, the alternative is the Latin-Greek hybrid "milli-micro" mlmc.

§trilli (tr) from trillionth, may be considered as an alternative to micro micro.

¶the Greek "penta" suggesting the exponent 5. A. E. Kennelly has suggested the prefix "lakh" from the Sanskrit for 100,000.

The Importance of Unencumbered Names for the Formula Units. Unvarying adherence to the principle that, "In forming the name of a s. l. a. m. unit from the name of the formula unit, a given decimal prefix, as "kilo," shall always signify the same multiple of the formula unit," means that it is not permissible to *encumber the names of formula units* with misleading decimal prefixes. To comply with this principle, new names will have to be coined and applied to four or perhaps five of the formula units which appear in the comprehensive systems which have been proposed for universal use. For example, in the c. g. s. systems, the name of the unit of length is encumbered with the prefix "centi;" in the m. k. s. system, the name of the unit of mass is encumbered with the prefix "kilo;" in the c. g. s. system, the name of the unit of mass is encumbered with the suffix "seven."

The formula units which must be disencumbered by renaming them, and the new names which I venture to suggest are shown in Table II.

Basis of Comparison of Systems of Units. To summarize, the desiderata in a comprehensive system of units for universal use are:

1. The formulas expressing the relations between quantities should not be *encumbered* with needless or perplexing numerical factors.

2. The *formula units* of the system should have names which are not encumbered with misleading decimal prefixes. The decimal prefix in a name should always signify that the unit is a s. l. a. m. multiple of the formula unit.

3. The *formula units* of the few quantities dealt with in the every-day affairs of community life should be readily comprehensible. In addition it is desirable that in the case of all the formula units of the system, their s. l. a. m. units lying between the micro and the mega unit shall be of convenient size to meet the needs of the shop, laboratory, and market.

TABLE II
PROPOSED DISENCUMBERED NAMES FOR THE FORMULA UNITS

	M. K. S. system		C. G. S. S. system	
Quantity	old name	new name	old name	new name
length......	meter	meter	centimeter	tyc*
mass........	kilogram	newt	gram-seven	newt†
force........	dyne-five	galileo or galeo	dyne-seven	galileo‡ or galeo
area.........	sq. meter or centare	derm	sq. cm.	derm§
volume.....	cu. meter or stere	stere	cu. cm. or milli liter	cuba¶

*To commemorate the industry, the skill, and the vision of Tycho Brahe, the measurer preeminent, who, before the inventions of the telescope, made the precise astronomical measurements from which Kepler was able to deduce the laws of planetary motion.

†To commemorate Newton's invention of the concept of "the mass of a body."

‡To commemorate Galileo's keen observations of the circumstances attending the motions of bodies.

§From the Greek "derma"—skin.

¶From the Greek "kybos"—cube.

Deficiences of the International System of Electric and Magnetic Units. In the light of this discussion of the basis for comparing the relative merits of different systems of units, let us first consider the *deficiencies* of the hybrid system which has been legalized throughout the civilized world by governmental actions, namely the centimeter, gram, second absolute system for the mechanical units and the international system of electric and magnetic units.

The fundamental formula units of this hybrid system are:

a. The unit of time is the second of the mean solar day; this unit is common to both the absolute and the international sets of units.

b. In the absolute set, the centimeter and the gram, as represented by the platinum-iridium prototype meter and prototype kilogram Standards, in the custody of the International Bureau of Weights and Measures, are the remaining two fundamental units.

c. In the international system, the two fundamental units, other than the second, are the ohm and the ampere as represented by the mercury ohm standard and the silver nitrate coulombmeter set up in accordance with the legal specifications for the international ohm and the international ampere.

This hybrid system of units is deficient in the following respects.

1. The international unit of energy, the joule, as represented by the energy expended per second when a current of 1 international ampere flows in a resistance of 1 international ohm is not equal to 1 absolute unit of energy, it is not even equal to 10^7 absolute units, or 10^7 ergs, but

$$\begin{aligned} 1 \text{ International joule } &= 1.00034 \times 10^7 \text{ ergs} \\ &= 1.00034 \text{ ``practical'' absolute joules.} \end{aligned}$$

This means that formulas for obtaining the mechanical forces and the accelerations in electrical systems are encumbered with the needless numerical factor, 10^7. For example,

$$f \text{ (dynes)} = 10^7 I_1 I_2 \frac{d\,M}{d\,x} \quad \frac{\text{(amperes, henries)}}{\text{(cm.)}}$$

$$a \text{ (cm. per sec. per sec.)} = 10^7 \frac{F\,q}{m} \quad \frac{\text{(volts per cm., coulombs)}}{\text{(grams)}}$$

2. The last international actions on the magnetic units are

the action of the International Electric Congress in Chicago in 1893 recommending the use of the unrationalized magnetic units of the electromagnetic system of c. g. s. absolute units and the action of the Fifth International Electrical Congress of Paris in 1900 recommending the use of the following names.

"*maxwell*" for the e. m. unit of magnetic flux.

"*gauss*" for the e. m. unit of field intensity.

The use of the "maxwell" for the unit of flux means that many formulas involving flux are encumbered with the needless numerical factor 10^{-8}. For example,

$$e\text{ (volts)} = 10^{-8}\,\frac{d\,\varphi}{d\,t}\ \frac{\text{(maxwells)}}{\text{(seconds)}}$$

$$L\text{ (henries)} = 10^{-8}\,\frac{\varphi\,N}{I}\ \frac{\text{(maxwells, turns)}}{\text{(amperes)}}$$

$$W\text{ (joules)} = 10^{-8}\,I\,N\,\Delta\,\varphi\text{ (amperes, turns, maxwells)}$$

but note the exception,

$$f_x\text{ (dynes)} = I\,N\,\frac{d\,\varphi}{d\,x}\ \frac{\text{(amperes, turns, maxwells)}}{\text{(cm.)}}$$

The use of the "gauss" rather than the "ampere-turn per cm." as the unit of field intensity means that the formulas must be written in the unrationalyzed forms, thus

$$\text{M. M. F. (gauss-cm.)} = 0.4\,\pi\,N\,I\text{ (turns, amperes)}$$

$$\varphi\text{ long-solenoid (maxwells)} = \frac{0.4\,\pi\,\mu\,N\,I\,a}{l}\text{ (amperes, cm.)}$$

A Common Feature of the Proposals to Modify the Existing Hybrid System of Units. The proposals to improve upon this internationally sanctioned hybrid system of units have one feature in common. This feature is the proposal to "rationalize" the formulas of the system, by shifting the irrational $4\,\pi$ factors from the more frequently used equations to the less frequently used equations.

But there are two radically different proposals as to the method of bringing about this rationalization. The first proposal is that of those physicists who still persist in writing in Heaviside units. Now the ratios of the Heaviside practical units to the corresponding units in the International system which is in universal use throughout the world are shown in Table III.

TABLE III
RATIO OF THE HEAVISIDE PRACTICAL UNITS TO THE CORRESPONDING INTERNATIONAL UNITS

Unit of	Ratio H/Int
Charge and current	$1/\sqrt{4\,\pi}$
E.m.f. and electric intensity	$\sqrt{4\,\pi}$
Capacitance	$1/4\,\pi$
Resistance	$4\,\pi$
Magnetic intensity	$1/\sqrt{4\,\pi}$
Magnetic flux	$\sqrt{4\,\pi}$
Dielectric constant	1
Permeability	1

In other words, writers who continue to write texts in Heaviside units must virtually be advocating that some future international congress shall take an action which will require that the scales of all ammeters in the world be regraduated for full scale readings equal to $\sqrt{4\,\pi}$ times their present indications, that all voltmeters scales be regraduated to read $1/\sqrt{4\,\pi}$ their present values, that all condensers be restamped with ratings equal to $4\,\pi$ times their present ratings, and that all resistance coils be restamped with $1/4\,\pi$ their present ratings.

Furthermore, this action will mean that all tables showing the dielectric strengths, the resistivities, and the hysteresis losses of materials must be revised, and that all B/H curves must be redrawn. And for what purpose? The answer is to be found in the last two lines of Table III. It is as follows: This stupendous job of recalibrating instruments and of revising tables is to be done in order that in the process of rationalizing the equations of the dielectric and the magnetic circuits, the tables of dielectric constants and of permeabilities may not be disturbed. And yet existing tables of these constants almost invariably give the *relative* dielectric constants and *relative* permeabilities of the materials, (relative to free space). Now tables of relative dielectric constants and of relative permeabilities will remain unaltered no matter what changes are made in the fundamental units. And so it transpires that the proposal to rationalize the formulas by changing to Heaviside units amounts to a proposal to change the current, voltage, capacitance, and resistance ratings of every ammeter, voltmeter, wattmeter, lamp, generator, transformer, condenser, and resistor in existence, for the sake of rationalizing the formulas by a process which will leave undisturbed the very tables which will remain unaffected by any change in the units.

The improbability of such an action by any standardization committee is so overwhelming that we may dismiss the proposal to rationalize the formulas by using Heaviside units without a moment's consideration.

In the second method the rationalization of the equations is brought about by two simple expedients.

1. By using as the units of magnetomotive force and magnetic intensity, the *ampere-turn* and the *ampere-turn per centimeter*, instead of the gilbert and the gilbert per cm. (gauss).

2. By drawing a distinction between the *dielectric constant* k and the *permittivity* p of a dielectric. The permittivity is related to the dielectric constant by the defining equation $k = 4\,\pi\,p$, and the permittivity is used in all equations.

By these simple expedients, the substance of all that Heaviside sought to accomplish by his so-called Rational System of Units is attained. This result is attained by submerging the objectionable $4\,\pi$ factors in the empirical constants (of materials) which the equations contain. The change does not require the rerating of any electrical device, it does not require the change of a single table or curve except B/H tables and curves, and tables which give the absolute values of the permeabilities and dielectric constants of materials in unrationalized practical units. The writer has yet to encounter such a table. The existing tables giving the relative permeabilities and the relative dielectric constants in the unrationalized system correctly express the relative permeabilities and the relative permittivities in the rationalized system. The B/H curves will have to be replotted, with H expressed in ampere-turns per cm. instead of gilberts per cm. The constants of free space in the unrationalized and rationalized international units are:

Unrationalized units	$k_o = \dfrac{1}{9 \times 10^{11}}$	$\mu_o = \dfrac{1}{10^9}$
Rationalized units	$p_o = \dfrac{1}{4\,\pi\,9 \times 10^{11}}$	$\mu_o = 4\,\pi/10^9$

Comparison of the Merits of the Alternative Systems of Practical Units. Dismissing from consideration the revolutionary and unwarranted proposal to supplant the International units with Heaviside units and postulating that the equations of the dielectric and magnetic circuit will be rationalized by the simple expedients sketched above, it remains to compare the merits for universal use of the alternative systems of practical units.

The term "practical system" designates (among other things) a system in which the unit of energy (the joule) is defined to be equal to 10^7 ergs and in which the unit of time is the second, but in which nothing has been said about the units of length and mass, except that with the units of time and energy fixed upon, the units of length, mass, and force must be selected from metric combinations like the following.

System	Units		
	Length	Mass	Force
....	millimeter	gram-nine	dyne-eight
C. G. S. S.	centimeter	gram-seven	dyne-seven
....	decimeter	gram-five	dyne-six
M. K. S.	meter	kilogram	dyne-five
....	dekameter	dekagram	dyne-four
	etc.	etc.	

One of these combinations must be used if such formulas as the following are to be kept free of needless numerical coefficients.

energy = 1/2 mass × velocity^2
work = force × distance

Of the possible practical systems shown above only two have been advocated for universal use, namely the centimeter, gram-seven, second (c. g. s. s.) system and the meter, kilogram, second (m. k. s.) system. If the names of the formula units in these systems are to be unencumbered, it will be necessary to coin new names for the units of length, mass, and force in the c. g. s. s. system, and for the units of mass and force in the m. k. s. system. Names for these units are suggested in Table II.

Any judgment as to the relative merits of the c. g. s. s. and the m. k. s. systems must be based upon a comparison of the expressions for the values of the quantities of frequent use when stated in the c. g. s. s. and m. k. s. units respectively. To furnish material for such a comparison, the following statements are offered.

EXPRESSIONS FOR THE SAME QUANTITY IN THE C. G. S. S. AND M. K. S. SYSTEMS

Expressions involving length	C. G. S. S.	M. K. S.
Unit........................	tyc	meter
Dimensions on machine drawings are in.................	decitycs	millimeters
The thickness of the paper is...	6 millitycs	0.006 centimeters
The dimensions of the planks are	4 x 24 x 480 tycs	4 x 24 x 480 cm.
Cloth is measured in...........	hektotycs	meters
The width of the street is......	15 hektotycs	15 meters
The distance between towns is..	5 pentatycs	5 kilometers
The New York-Chicago distance is.........................	1150 pentatycs	1150 kilometers
The train speed was..........	80 pentatycs per hr.	80 kilometers per hr.
The speed of light is...........	3 x 10^{10} tycs per sec.	3 x 10^8 meters per sec.
The mean spacing air molecules is....................	0.33 microtycs	3.3 x 10^{-9} meters
The diameter of the oxygen molecule is.................	3 x 10^{-8} tycs	3 x 10^{-10} meters
Expressions involving area		
Unit........................	derm or sq. tyc	derm or sq. meter
The sectional area of No. 10 B & S wire is...............	5.26 centiderms	5.26 microderms
The sectional area of the beam is	20 sq. tycs or 20 derms	20 sq. centimeters or 20 demilderms
The area of the 20 x 50 lot is....	1000 sq. hekto tycs	1000 sq. meters or 1000 derms
1 acre equals.................	0.405 sq. myria tycs	0.405 sq. hekto meters
Expressions involving volume		
Unit........................	cuba or 1 cu tyc	stere or 1 cu. meter
The milk bottle capacity is.....	1 kilo cuba	milli stere
The truck capacity is..........	2 mega cubas	2 steres
The reservoir capacity is.......	50 million mega cubas	50 million steres
Expressions involving mass		
Unit........................	newt = gram-seven	newt = kilogram
Butter packages contain.......	0.5 demil newts	0.5 newts
The man's weight is...........	70 demil newts	70 newts
The coal costs $5.00 a.........	decinewt	kilonewt
The allowable truck load is.....	5 decinewts	5 kilonewts
The mass of an electron is......	9 x 10^{-35} newts	9 x 10^{-31} newts
Expressions involving density		
The density of water is.......	0.1 micronewts per cubic tyc	1. kilo newt per cubic meter

EXPRESSIONS FOR THE SAME QUANTITY IN THE C. G. S. S. AND M. K. S. SYSTEMS—*Continued*

Eqpsessions involving length	C. G. S. S.	M. K. S.
Expressions involving force		
Unit........................	galeo = dyne^7	galeo = dyne^5
The pull of the earth on an apple is.....................	1 centi galeo	1 galeo
The pull of the earth on a newt is	980 galeos	9.8 galeos
A man can exert a lifting pull of	700 centigaleos	700 galeos
Normal atmospheric pressure is	10.132 centigaleos per sq. tyc	0.10132 mega galeos per sq. meter
300 lbs. boiler pressure is......	200 centi galeos per sq. tyc	200 galeos per sq. cm.
The tensile strength of steel is..	500 galeos per sq. tyc	500 mega galeos per sq. meter
The modulus of elasticity of steel is....................	207,000	2.07 x 10^{11}
Gravitational constant........	0.666	6.66 x 10^{-11}
Coefficient of viscosity of water, 20 deg. cent...............	10^{-9} galeos, tycs	10^{-3} galeos, meters
Gas constant R in $pv = Rt$.....	3.718 x 10^{-4} galeos, tyc, deg. cent.	3.718 x 10^2 galeos meter deg. cent.
Expressions involving heat		
1 newt deg. cent..............	4.187 x 10^7 joules	4.187 x 10^3 joules
Tables of specific heat will not be affected		
Thermal conductivity of copper in watts per unit area per degree per unit length.........	3.88	388
Ditto for wood...............	0.0004	0.04
Electric and magnetic expressions		
Permittivity of space..........	8.85 x 10^{-14}	8.85 x 10^{-12}
Permeability of space..........	4π x 10^{-9}	4π x 10^{-11}
Resistivity of copper..........	1.724 x 10^{-6} ohm, tyc	1.724 x 10^{-8} ohm, cm.
Dielectric strength of air.......	30,000 volts per tyc	3 megavolts per meter
Flux density in transformers...	120 micro webers per sq. tyc	1.2 webers per sq. m.

S. L. Gokhale: The paper under review gives a concise and very clear history of the problem of magnetic nomenclature which has inspired so much discussion during many decades in the past. This historic sketch gives a good perspective view of the whole situation, and is helpful for a clearer understanding of the true nature of the controversy. It has certainly helped me, or at least I believe it has, for which I am very thankful.

There are some points not clear to me until now; for example, I could not see why magnetic induction is expressed by the symbol B. In my college days, I was taught to look on magnetic flux as something analogous to emanation of light from a lamp. I had found that analogy very helpful in that it gave me a clear conception of the equations of Gauss, Poisson, and La Place, but it did not occur to me until I read Dr. Kennelly's paper, that magnetic induction (B) is analogous to optical brightness at a point, the latter being also expressed by the symbol B. I should like to be informed, however, if that is the true explanation.

The problem of nomenclature resolves itself into three separate but closely related problems:

(a) the relation of the several units to each other and their logical sequence.

(b) Size of the units.

(c) Names of the units.

The question of name is obviously the simplest and may be taken up first.

The greatest amount of controversy ranges itself around an appropriate name for the unit of magnetic force intensity (magnetizing force, or magnetic field intensity) expressed by the symbol H. Three names have been used in the past, and four more are under consideration namely:

A; names in usage
A—1, Gilbert per centimeter
A—2, Gauss per unit permeability (briefly gauss)
A—3, Dyne per unit pole.

B; names proposed

- B—1, Gauss (Paris Congress; Delegation for Holland and Italy)
- B—2, Mascart (French delegate)
- B—3, Weber (German delegate)
- B—4, Gilbert (Holland delegate), for practical unit only.

The four "proposals" have all the merit of being simple names instead of the three complex names in "usage." Among these three, the weight of usage seems to be greatly in favor of the term "gilbert per centimeter."

While popularity of the name may well be conceded, it must also be conceded that it lacks the simplicity and the convenience of a short single worded name, justifying the use of an abbreviated substitute. In view of this fact I had decided to use in the records and reports of my work the single letter "g" as an alternative name of this unit, and have consistently done so for the last six years. (A. I. E. E. 1926, *Law of Magnetization*; 1928, *Saturation Permeameter*). It is necessary to add however, that although the use of the name "g" was technically a mere abbreviation of the term "gilbert per centimeter," the change was guided in my mind by a more important consideration that goes to the root of the problem.

The paper under discussion together with the discussion which followed the reading of it at the convention, brings out very clearly the indisputable fact, that there are among the scientists of the world, two different conceptions as to the nature of magnetic flux. For convenience of discussion, I shall refer to them as the static and the dynamic conceptions.

According to the static conception of flux, magnetic induction, or flux density as it is sometimes called, is a magnetic force of exactly the same kind as the magnetizing force H, and differs from it only in the location of the sources from which it emanates. The total or resultant magnetic force at a point in a magnetic body in general, is made of two parts:

(a) Resultant of forces emanating from distance sources, such as magnetism on the free surface of the body, magnetism on surfaces of neighboring bodies, and electric currents magnetically associated with these bodies. This is called the magnetizing force H.

(b) Resultant of forces emanating from sources infinitely near the point, such as the magnetic poles of molecular magnets (Weber's theory), or molecular electric circuit (Ampère's theory) or the microcosmic electronic orbits, (Langevin's theory) in the neighborhood of the point. This force is called intrinsic force or intrinsic induction β.

According to this definition $\beta = 4\pi J$, where J is the intensity of magnetization. The total magnetic force at the point is called total induction or briefly the induction B. Thus we have the relation $B = \beta + H = 4\pi J + H$. The method of computing analytically the forces from the near and distant sources, by the use of an auxiliary imaginary cavity of proper size, shape, and orientation, is described in all leading text books and need not be repeated here. (J. J. Thomson: "Electricity and Magnetism," Art. 153) According to this conception B and H have identical dimensions and therefore, permeability (μ) must have zero dimensions, with the value $\mu_o = 1$ for free space.

According to the dynamic conception of flux, magnetic flux is analogous to electric current, while the magnetizing force corresponds to electromotive force (Karapetoff, "The Magnetic Circuit," p. 5). The analogy is best expressed by the equations.

$$\frac{B}{H} = \mu \text{ (Permeability) } ; \frac{I}{E} = \gamma \text{ (conductivity)}$$

The dimensions of μ are supposed to be unknown but not zero; its value for space may be taken as unity. According to this conception, induction and magnetizing force are dissimilar quantities, which call for different units for their respective measurements. The static conception is generally favored by physicists (J. J. Thomson: *Ibid.*, Art. 256 footnote), and the dynamic conception by the electrical engineers, (Karapetoff, *Ibid.*, p. 257) perhaps because each group finds his favorite system better adapted for his special work. Theoretically there is nothing to guide us in our choice at present, and the situation is not likely to be changed in any way for some years to come.

In view of the facts, I suggest, that:

(a) There should be two systems of nomenclature.

(b) One of these should be based on the dynamic conception of flux having separate units for magnetizing force and flux; the terms "gilbert per centimeter" and "gauss" might be accepted as their most convenient names.

(c) The other system should be based on the static conception, and should have one single unit for both force and flux, and of course with a single name.

(d) It should be recognized that eventually one of the two systems will have to be discarded, but we do not know which one it is to be. The names of the units should be so selected that either system could be easily discarded at any time if necessary, without inconvenience to the readers and users of magnetic literature and data in which the discarded system will have been used.

(e) The unit "g" meets all these requirements. As an abbreviation for "gilbert per centimeter" and also for "gauss" if necessary, it is capable of replacing both of them, if the dynamic system is ever discarded. On the contrary if the static system is discarded the letter "g" may still retain its usefulness as a mere abbreviation of "gilbert per centimeter" only.

The unit magnetic pole, is another quantity that deserves a name. If, as some scientists advocate, we decide in favor of discarding the conception of poles altogether, there will be of course no need for a name; but if we intend to use it in the future, as much as we have been using it in the past, it certainly ought to have a unit with a name worthy of its position. I suggest the name "ewing."

The need of a convenient name for the practical unit 10^8 maxwell, has long been felt, but no acceptable name has yet been suggested. The name "line" as a synonym for maxwell has long been in use and is very popular; the names "kilo-line" and "mega-line" as convenient names for 10^3 maxwell and 10^6 maxwell respectively, have also become quite popular. In view of these facts I suggest the name "arb-line" as a synonym for 10^8 maxwell, (the prefix "ARB-" being a part of the Sanskrit numeral "arbuda" which means "hundred-million") with a confession of my ignorance of Latin and Greek as an apology for the suggestion; the convenient form of the word is its own defense.

The name "electromagnetic" as applied to that well known system of units seems to be a misnomer. In contra-distinction to the "electrostatic," the name "magnetostatic" seems to be more appropriate; I have used it in my own records and reports even without the formality of an explicit statement to that effect, and have had no protest. The following are formal definitions of some of the terms, as I use them in my analysis of systems of units:

An *electromagnetic system* of units, is a system of both the electric and the magnetic units; it includes the "electrostatic" system.

An *electrostatic system* is a complete "electromagnetic" system involving the dual assumption $K_o = 1, \mu_o = 1/V^2$ with or without any factors of convenience.

A *magnetostatic system* (popularly called electromagnetic) is a complete electromagnetic system involving the assumption $K_o = 1/V^2, \mu_o = 1$, with or without any factors of convenience.

A *comprehensive system* is a system in two parts, with electrostatic units for the electric part and with magnetostatic units for the magnetic part, the two parts being connected by the equation $\mathfrak{F} = I/V$, with or without any factors of convenience.

(NOTE: Dr. Kennelly has used the word "comprehensive" in another sense.)

A *composite system* is a system involving the dual assumption that $K_o = 1/V$, and $\mu_o = 1/V$ with or without factors of convenience.

The "factors of convenience" referred to in the above definitions, are usually factors of the form $(4\pi)^m\, 10^n$, where m and n have any convenient values.

For tabular classification of these systems see Table II of this review. Referring to Dr. Kennelly's paper, (Table II, units No. 33 and 37), it should be obvious, that all the four systems are magnetostatic, and unified. None of them is comprehensive in either sense. The only difference between them is the sizes of the units, due to differences in sizes of the fundamental units to start with. Leaving aside for a time the whole question of 4π, and limiting the discussion to the question of the factor 10^n, the difference between the C. G. S. system (magnetostatic) and the three new systems (also magnetostatic) now submitted for consideration, can be reduced to this, namely, that until now it has been our practise to call a "volt" a practical unit and a "gram" an absolute unit, but if we adopt any of the three proposed systems we shall from now on, call a "volt" an absolute unit, and a "gram" a practical unit. This is about all that can be said of the nature of the differences between the old and the new systems. So far as practical use is concerned, we shall use the volt and the gram in the future exactly as we have been doing in the past, irrespective of their being classified as "absolute" or "practical." Concerning this war of the units, one might paraphrase the famous Battle of Blenheim:

> It is indeed a glorious war,
> But what are they fighting each other for?

Let us now take up the question of 4π in the equation $F = 4\pi N I/10$. This factor has been challenged on grounds of theoretical invalidity, and of practical inconvenience. The theoretical aspect of the question takes several forms, of which two are most important.

(a) The presence of 4π in the equation $\mathfrak{F} = 4\pi N I/10$ is due to the mode of derivation of the equation from the polar conception of magnetism, through the intermediate conception of magnetic shells. The polar conception of magnetism is a pure scientific fiction; it is now abandoned by the modern scientific world. The equation has therefore lost its theoretical foundation. It has now no other foundation except the experimental fact that $\mathfrak{F}$ is proportional to $N I$; there is no reason for assuming any constant in particular. The simplest legitimate assumption is $\mathfrak{F} = N I$, which now becomes an equation for defining $\mathfrak{F}$ in terms of I.

(b) The equation $\mathfrak{F} = 4\pi N I/10$ is mathematically related to the equation $B = H + 4\pi J$. The latter equation is also based on the polar conception of magnetism, and moreover it involves the twofold assumption that permeability of space is $\mu_o = 1$, and has no dimensions. This assumption is unwarranted and contrary to the probability of the case, which makes the 4π in both the above equations invalid.

The contention, that "magnet poles are purely fictitious," is itself a fiction which is best refuted by the authors of that contention. For example, according to Professor Karapetoff (*Ibid.*, p. 263)

> "The concept of pole strength is of no use in electrical engineering, and in the author's opinion, its usefulness in physics is more than doubtful. The whole theory of electromagnetic phenomena can and ought to be built up on the two laws of circuitation. . . ."

A reference to pages 2 and 3 of the same book will show that he finds an important use of magnet, pole, *viz.*, to demonstrate the existence of the lines of magnetic force; nor is this merely a case of professorial license, because as far as I know there is no other way of proving the existence of these lines. If magnetic poles be consigned to the realm of fiction, the lines of magnetic force go there also, and lines of magnetic flux also follow, in their turn. One important reason for doubting the existence of poles is, that we sometimes meet with pole-less lines, that is, closed magnetic lines circling round a wire carrying current. But if the existence of such pole-less magnetic lines is an evidence of non-existence of any magnetic poles, the existence of similar endless lines of electrostatic force should also prove the fictitious character of electric charges. Another reason for assuming that the magnetic poles are fictitious is, that every kind of magnetic field can be construed as the result of certain hypothetical, well designed electric current circuits, that magnet poles are therefore not necessary for production of magnetic fields and equipotential surfaces, and that they should be therefore looked upon as mere scientific fiction. The argument is far from convincing. First, even if every function of a magnetic pole were fulfilled by a system of electric currents, that could not prove the poles to be non-existent; and secondly every theoretical function of a magnet pole cannot be fulfilled by electric current. It can be shown that every magnetic or electromagnetic system can be represented in theory by a series of harmonic magnets, the first member of the series being a single magnet pole, which gives rise to a potential function of the form of simple spherical harmonic of zero order (*Magnetic Shielding*, A. I. E. E. TRANS., Oct. 1929). Theoretically, no conceivable single electric circuit can take the place of this pole or produce the corresponding spherical harmonic. Practically, I know of no electric device to demonstrate the existence of magnetic lines of force circulating round a wire carrying a current. See Fig. 1.

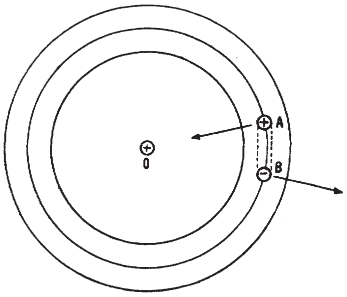

FIG. 1

Let o be a wire perpendicular to the plane of the paper, carrying a current from the front to the back side, and producing the circular lines of force whose existence is challenged. $A\,B$ is a projection of a small electric circuit parallel to the wire o, and parallel also to the magnetic field produced by o. According to Ampère's law relating to action of a current on a current, the part A approaches o and the part B recedes from it. These forces are radial. There is no tangential force; there is therefore neither a magnetic field nor a magnetic flux. This is the conclusion we reach when we ignore magnetic poles and try to interpret magnetic phenomena in terms of electric currents. Since it is not possible to ignore magnetic poles without ignoring electromagnetic fields, any argument against the use of 4π (in the basic formula $F = 4\pi N I$) based on the alleged fictitious character of magnet poles, falls to the ground.

The argument based on the theory of dimensions of permeability is equally fallacious. First, the assumption that permeability of space is $\mu_o = 1$, has nothing to do with dimensions of μ. The dimensions of density are M/L^3, but that does not prevent us from assuming $D = 1$ for water in physical measurements or for hydrogen in chemical measurements; second, the equation $B = H + 4\pi J$, may or may not be valid; it depends on the purpose of the equation. Dr. Karapetoff's argument, "that we cannot have sheep on one side and apples on the other side of an equation" is sometimes valid and sometimes not. Suppose that I kill by accident one of Professor Karapetoff's sheep, and send him by way of compensation one barrel of apples and in addition a check for \$12-57/100; in my memoranda I write, one sheep

= one barrel apples + 4 π dollars. What is wrong with the equation? Thirdly, the contention that μ must have some dimensions other than zero, is far from proved. If B and H be defined in accordance with the static-conception, their dimensions must necessarily be identical, leading to zero dimensions for μ as a necessary logical consequence. If on the contrary B and H be regarded as analogous to I and E, and defined in accordance with the dynamic conception, then μ becomes analogous to R and must have some dimensions other than zero. The question of dimensions of μ is not yet settled and is not likely to be settled in the near future, because the question involves three physical characteristics of space, *viz.*, K, μ, and P with only one physical equation connecting them. (See Table I, Equation **5**.)

This brings us to the question of physical characteristics of space-medium involved in electric, magnetic, electromagnetic, and magnetoelectric phenomena. These phenomena can be expressed by the five fundamental equations.

TABLE I

FUNDAMENTAL EQUATIONS OF ELECTRIC AND MAGNETIC PHENOMENA

(1)	$e^2/K L^2 = f\ M\ L/K\ T^2$	Electrostatic relation (Coulomb's Law:electric)
(2)	$m^2/\mu\ L^2 = f = M\ L/\mu\ T^2$	Magnetostatic relation (Coulomb's Law:magnetic)
(3)	$F = H\ L = I\ .\ a/P$ $= e/T\ .\ a/P$	Electromagnetic relation (Ampère's Law)
(4)	$E = \phi/T\ .\ b/P$	Magnetoelectric relation (Faraday's Law)
(5)	$P^2/K\ \mu = V^2 = L^2/T^2$	Electromagnetic propulsion (Maxwell's Law)

Notes: (1) a and b in Equations 3 and 4 are factors of convenience; according to the prevailing practise at present we have $a = 4\pi$, and $b = 1$. In some of the recently proposed systems, we shall have $a = 1$, and $b = 1$; these assumptions are merely matters of convenience.

(2) The symbol P in Equations 3, 4, and 5, is the same as λ of Ascoli (*Elec. World*, Vol. 67, p. 877) or as c of Heaviside (*Bul. B. of S.*, Vol. XIII, p. 613). No name for this characteristic has yet been suggested. I suggest the name "propulsivity." The symbol P was introduced by J. J. Thomson.

These laws involve three physical characteristics of the medium which in three ways contributes towards the electromagnetic phenomena, *viz.*:

K, or dielectric character of the medium; it determines the magnitude of the electrostatic force between two electric charges at rest.

μ, or permeability of the medium; it determines the magnitude of the magnetostatic force between two magnet poles at rest.

P, or propulsivity of the medium; it determines in the first place the magnitude of the kineto-static force produced by a moving electric charge on a magnet pole at rest, or by a moving magnet pole on an electric-charge at rest. As a further consequence it contributes to the determination of the velocity of propulsion of electromagnetic impulses in space. When P is assumed to be equal to unity (with or without other factors of convenience) the equation of propulsion takes the form $V^2 = 1/K\ \mu$, and it then appears as if K and μ are the only factors involved in the process. This is also the reason why the probability that "the dimension of μ is not zero" appears to be so high. (Dellinger, *Bul. B. of. S.*, Vol. XIII, p. 610-611). The root of the whole difficulty lies in the fact generally unrecognized that μ and K are both similar and dissimilar. Their dimensions if they have any, must be therefore complementary. This is the only way in which they could be both similar and dissimilar. But in that case, they would be incapable of yielding a product such as $V = L/T$. In the Heaviside system, it is assumed that $\mu_o = 1$, $K_o = 1$, and $P = V = L/T$. This seems to be the most reasonable assumption. Anyway, the dogma that μ has dimensions other than zero, has nothing to support it except another dogma that P is not a physical characteristic, and that the velocity V is dependent on μ and K only. (Dellinger, *Bul. B. of S.*, Vol. XIII, p. 620.)

Since the three quantities K, μ, and P are related by only one known equation, there is ample room for speculation and arbitrary assumption, which leads to various systems of units, differing fundamentally, from each other, without going into the question of 4 π or of 10^n. See Table II.

TABLE II

SYSTEMS OF ELECTROMAGNETIC UNIT

System	Defining assumption		
Electrostatic	$K = 1;$	$P = 1;$	$\mu = 1/V^2$
Magnetostatic (popularly called electromagnetic)	$\mu = 1;$	$P = 1;$	$K = 1/V^2$
Comprehensive (Heaviside)	$\mu = 1;$	$K = 1;$	$P = V$
Composite-general	$\mu = 1/V;$	$K = 1/V;$	$P = 1$
Composite-particular type (*e. g.*, Karapetoff)	$\mu = R/V:$	$K = 1/RV;$	$P = 1$

We are now in a position to classify the various systems under discussion in accordance with the above definitions.

TABLE III

CLASSIFICATION OF THE SEVERAL SYSTEMS (OMITTING 4 π, 10^n, R, etc.

Name of system	Class	Defining conditions		
		μ	K	P
(1) C. G. S. (Electromagnetic)	Magnetostatic	1	$1/V^2$	1
(2) Q. E. S. Maxwell (Kennelly, Table III)	Magnetostatic	1	$1/V^2$	1
(3) Giorgi (Kennelly, Table III)	Magnetostatic	1	$1/V^2$	1
(4) Giorgi (*B. of S.*, Vol. XIII,p.619)	Composite	$1/V$	$1/V$	2
(5) International (Dellinger & Bennet) (Kennelly, Table III)	Magnetostatic	1	$1/V^2$	1
(6) International (Dellinger) *B of S.*, Vol. XIII, 603)	Composite	$1/V$	$1/V$	1
(7) Bennett & Crothers ("Electrodynamics," p. 639)	Magnetostatic	1	$1/V^2$	1
(8) Karapetoff ("The Magnetic Circuit," p. 263)	Composite	$1/V$	$1/V$	1

It should be obvious from the above, that all of the systems now under discussion (*viz.*, 1, 2, 3, and 5) involve the assumption that $\mu = 1$ irrespective of dimensions. Therefore the case for the removal of 4 π, based on the consideration of possible dimensions of μ, falls to the ground, in so far as Dr. Kennelly's paper is concerned.

Another theoretical argument for elimination or rather for dislocation of the 4 π from the equation $\mathfrak{F} = 4\pi\ N\ I/L$, is that it is irrational "by reason of the occurrence of 4 π as a factor in the specification of quantities which have no obvious relation to circles or spheres." This rational principle when applied to the equation in question, leads to the conclusion that the 4 π should be just where it is. For we have, first, the experimental fact that the force at a point at distance r from a straight wire is $H = \int I\ d\ l \sin\theta/r^2 = 2\ I/r$. Second; we have the inferential fact that the line integral of $H\ .\ d\ l$ round the wire is $4\pi\ I$, the factor r being eliminated in the process of this second integration. The factor π enters into the equation for the obvious reason that the magnetomotive force is a line integral round a circular path. The elimination of the distance r is due to the fact that it (r) acts in two opposite ways and cancels itself; it tends to diminish the intensity of the force, but it also tends to increase the length of the path in the same proportion. On account of the simplifications of the formula thus obtained, and also on account of the generalization it stands for, there has grown up a belief that the equation $\mathfrak{F} = c\ .\ I$ is a fundamental experimental fact, and that the equation $H = d\,\mathfrak{F}/d\,l = c\,.\,I/L$ is merely a mathematical inference as a secondary consequence. The fact is just the reverse; the second equation is the one experimentally determined, from which the first is obtained by integration (Bennett and Crothers: "Electrodynamics," Sect. 245).

The c. g. s. magnetostatic system of units is based on the legitimate assumption, that $d\,H = c'd\,l \sin\,.\,\theta\,.\,I$ is the fundamental equation and that its simplest form as obtained by assuming $c' = 1$, should be made the basis of the definition of I in terms of H or vice versa. From this equation we get by integration firstly, $H = \dfrac{2\,I}{r}$ for single straight wire, and secondly $\mathfrak{F} = 4\,\pi\,I$ for the complete path round the wire. The $4\,\pi$ in this equation is not irrational; it comes in for the simple and obvious reason, that it concerns a law of circuitation; this is exactly in accordance with the principle implied in the dictum of Oliver Lodge which disapproves of the "$4\,\pi$ as a factor in the specification of quantities which have no obvious relation with circles or spheres." (Heaviside, "Electrical Papers," Vol. II, art. 53, p. 575.)

TABLE IV
CHART SHOWING EVOLUTION OF MAGNETOSTATIC SYSTEM OF UNITS, WITH C. G. S. UNITS FOR *L*, *M* AND *T*

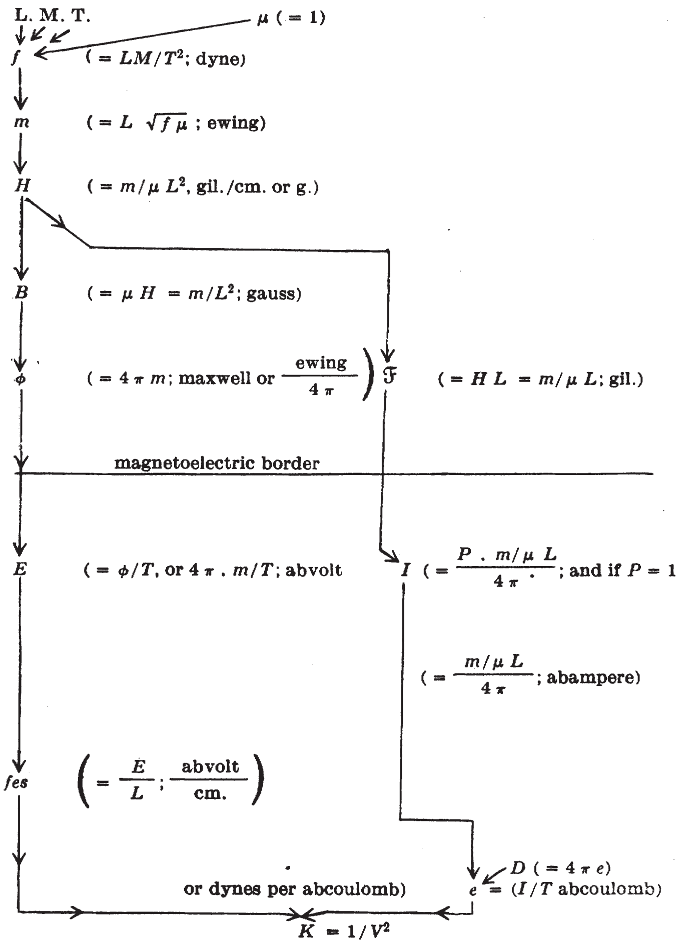
L. M. T.
$\mu\ (= 1)$
f $(= LM/T^2$; dyne)
m $(= L\ \sqrt{f\,\mu}$; ewing)
H $(= m/\mu\,L^2$, gil./cm. or g.)
B $(= \mu\,H\ = m/L^2$; gauss)
ϕ $\left(= 4\,\pi\,m\text{; maxwell or } \frac{\text{ewing}}{4\,\pi}\right)$
$\mathfrak{F}$ $(= H\,L\ = m/\mu\,L$; gil.)
magnetoelectric border
E $(= \phi/T$, or $4\,\pi\,.\,m/T$; abvolt
I $\left(= \frac{P\,.\,m/\mu\,L}{4\,\pi}\right.$; and if $P = 1$
$\left(= \frac{m/\mu\,L}{4\,\pi}\right.$; abampere)
fes $\left(= \frac{E}{L};\ \frac{\text{abvolt}}{\text{cm.}}\right)$
or dynes per abcoulomb)
$D\ (= 4\,\pi\,e)$
$e = (I/T$ abcoulomb)
$K = 1/V^2$

Table IV is a chart showing the evolution of the derived units of the magnetostatic system in their logical sequence. It shows that between the points m and e, there are only two equations involving circuitation, and therefore offering legitimate room for $4\,\pi$. Of these two, only one involves actual mathematical computation, and therefore brings in the $4\,\pi$ in manifest form. The other equation $E = \phi/T$ is derived directly by experiment, not indirectly by circuitous computation from an experiment. Therefore there is only one place for the rational appearance of $4\,\pi$ in the whole system between m and e. Of course there must be a $4\,\pi$ in the equation for E also; it is merely suppressed and can be made manifest by substituting $\phi = 4\,\pi\,m$. The true form of the equation is $E = \phi/T = 4\,\pi\,m/T$ where m/T is the magnetic current. The proposal to omit the $4\,\pi$ from the equation for $\mathfrak{F}$ is merely an irrational effort to ignore the indisputable fact, that the laws of circuitation involve a real circuitation. It may be suppressed by expressing I in terms of the polarization D, so that $\mathfrak{F} = D/T$, is analogous to $E = \phi/T$. This mode of eliminating $4\,\pi$, is rational but it is not likely to be popular.

From the purely practical point of view, it must be conceded that the presence of the factor $4\,\pi$ in the equation $\mathfrak{F} = 4\,\pi\,N\,I/10$ is a great inconvenience for the electrical engineer, and a very unnecessary one, because he is mostly concerned with ampere turns. He has absolutely no use for the hypothetical force of so many dynes per unit pole located hypothetically in an imaginary cavity within his magnetic material. This is, however, just the thing the physicist has to deal with. He is not satisfied with average value of H for the whole circuit,—he wants to know the local value of this force at any point. He is like a pupil in a swimming class who wants to know the local depth at the spot he is interested in; he refuses to trust his life to data on the average depth of the whole lake. The best solution of this conflict between the engineer and the physicist would be the use of a supplementary practical unit "the ampere turn," for the use of the practical engineers with or without a new name as they choose. The unit "gilbert per centimeter" briefly "g" should be left unchanged for the use of the physicist. The equation $B = H + 4\,\pi\,J$, (or $B = H + \beta$ as I prefer to express it) has done a tremendous service in the past; it has facilitated the work of Ewing, Kennelly, Steinmetz, and many others, and the possibility of its further usefulness in future cannot be discounted dogmatically. If the physicist appreciates the equation so highly and is willing to pay for the service by tolerating the $4\,\pi$ in all his computation of H let him do so.

In concluding, may I quote the words of Dr. Kennelly, "My personal opinion would be, that we had better leave well enough alone at the present time, and not upheave all our standards, our ohms, volts, and amperes for the sake of $4\,\pi$" (International Congress, St. Louis 1904, Trans., Vol. I, p. 179).

It is true that this opinion was expressed 26 years ago, but it is equally true, that science has made no new contribution during this interval to change the status of the $4\,\pi$. His reasons for the change are not clear from his presentation of the case. I must therefore hold to my conclusion, that in the equation $\mathfrak{F} = 4\,\pi\,N\,I$, the factor $4\,\pi$ should be allowed to stay just as it is, but that in response to the need of the electrical engineers, the unit "ampere turn" should be formally recognized as a supplementary practical unit.

G. W. O. Howe: (communicated after adjournment) After a careful reading of this paper, I have found nothing to dissent from, with the exception of one small matter concerning magnetic potential. The various alternative systems of units with their respective advantages and disadvantages are very clearly set forth. The British Association's Committee appointed at Oxford has not been able to make much headway, because we find ourselves divided into two camps—one containing those that regard H and B as of the same nature, and the other those who, like myself, regard H as the m. m. f. per unit length producing B. If one writes $B = H + 4\,\pi\,I$, then one must belong to the first camp (I = intensity of magnetization). This, however, is a waste of a good symbol to no purpose and it would be better to write $B = B_o + 4\,\pi\,I$ and leave H to designate the magnetizing force which, in the opinion of many, is required to produce a flux density B_o in air and B in any other medium such as iron. Those who do not believe in the existence of such a force need not then use H at all. This is really the crux of the whole question. Those responsible for the relevant section of the Glossary of Terms and Definitions issued by the B. E. S. A. do not regard H as a magnetizing force equal to a m. m. f. per unit length, but maintain that such a force is an unnecessary figment of the imagination, and that all we are justified in assuming is the magnetic flux density H in air, increased to B in iron, which exists, not as the result of any force acting at a point, but as the result of a current somewhere. I should not object to this point of view if those who favor it would agree that, as H and B are of the same nature and only differ in magnitude, it is unnecessary and confusing to use two different symbols for them when a simple suffix would suffice to distinguish them. It is confusing because most electrical engineers use H for a different conception; *viz.*, m. m. f. per cm.; whether this has any real existence as a magnetizing force acting at a point or is merely a convenient aid to magnetic calculation is immaterial to the point at issue.

I note that on the third page of the paper, column 2, it is said that "In America the custom has been to use the *gilbert* for the working unit of magnetic potential." I am sorry to see magnetic potential introduced as I have always found it a very troublesome conception. It is not a fundamental concept but a mathematical derivative. I regard the gilbert as the working unit of m. m. f. To illustrate my point, consider a uniformly wound toroid carrying a steady current. There is a m. m. f. of $(4\,\pi/10)\;I\;T$ acting around the magnetic circuit, producing at every point in it a magnetizing force H equal to the m. m. f. per cm.,—whether as a reality or as a mathematical abstraction is immaterial—but can any one say that there is any difference of magnetic state between any two points in the ring? Can anyone say of the two points A and B in the accompanying sketch that one is at a higher or lower magnetic potential than the other? I think not. Whatever reasoning leads to the conclusion that A is at a higher potential than B also leads to the conclusion that B is at a higher potential than A. I am forced to the conclusion that all points of the ring are at the same potential, however many gilberts are acting around it. I think it would be more nearly correct to say that the potential theory which was based on electrostatics and magnetostatics is inapplicable to the magnetic circuit.

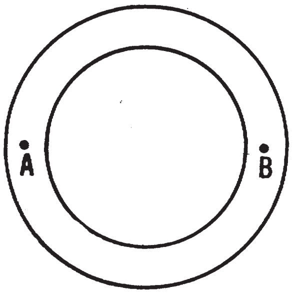

FIG. 2

B. Hague: (communicated after adjournment) I desire to express my appreciation of Professor Kennelly's very interesting paper, which focuses our attention upon the far from satisfactory condition of our magnetic units. I am very much interested in Dr. Kennelly's clear analysis of the various systems of units and his discussion of the question of nomenclature, but I wish to confine my remarks to what appears to me to be of even greater importance, the matter of definitions. I agree that before any international agreement on systems of units and nomenclature is attempted it is of primary importance to secure internationally acceptable definitions of magnetic quantities. It is surprising to discover the extent to which physicists and engineers disagree in regard to such fundamental things as the definition of H, B, and particularly μ; this disagreement is usually founded in misunderstanding. I would suggest that the time is now ripe for a thorough discussion of these matters by an international congress of engineers and physicists, and I think it would be possible to establish the basis for a set of definitions that would be rational, founded upon experimental fact and dependent upon hypotheses to the minimum extent. Much interest in this subject has already been shown in England, France, and Germany, where recent discussion of the main points of difference would be found, I believe, to have cleared the way already for the profitable international exchange of opinions and perhaps for some useful recommendations.

In framing a set of magnetic definitions I think it is essential to avoid all reference to "unit poles" and "cavities," left over from the old magnetostatics, and to realize that we are really concerned with electromagnetism,—a curl field phenomenon. Everything entering into the definitions should be of a measurable character; they should not be dependent upon conceptions, such as isolated unit poles situated in variously shaped cavities, which have no physical reality. Thus magnetic flux density (B) should be based upon a search-coil experiment and Faraday's laws; unit current should be defined from Ampère's laws for the mechanical force experienced by two circuits when carrying the current in series. Magnetic flux should be regarded to exist in a magnetic circuit when current-turns are applied to the circuit; induction or flux density at any point of the path of a magnetic circuit may be regarded, in the way usual to engineers, as due to the application at the point of a certain number of current-turns per unit length of the path. It would be a necessary consequence of a system of definitions of this kind to require the specification of the permeability of vacuous space μ_o, expressing the quantity with dimensions by which a certain number of current-turns per unit length must be multiplied to give the value of the induction B_o (often wrongly called H) in vacuo and the additional statement of specific permeability—a numeric—denoting the number of times B_o is augmented to B when vacuous space is replaced by a given material.

If we can decide how we shall define our magnetic quantities, and I desire to make a strong plea for some such experimental basis as I have indicated, the decision should go far to settle the best system of units and the problem of nomenclature. In the interest of uniformity and clarity of thought I would respectfully suggest that the I. E. C. invite discussion and agreement upon the major question of definition before taking up the choice of units and their names.

G. S. Gibbs: (communicated after adjournment) In my opinion the names maxwell, gauss, oersted, etc., are superfluous when applied to these units. No names for them are needed. As a rule an author need not even indicate whether he is using the C. G. S. or the practical unit. Misunderstanding will seldom, if ever, arise if no such indication is made. However, if an author considered it necessary it would complicate matters less to designate the unit as C. G. S. or practical than to attempt the use of any system of names.

Cecil L. Fortescue: (communicated after adjournment) This characteristically fair and unbiased statement of a complicated matter brings out the two essentials that have to be dealt with before any question of nomenclature can be settled, *viz.*:

a. To decide upon the definitions of the quantities themselves.

b. To determine whether there are, in fact, any grounds for departing from the C. G. S. system and its practical decimal multiples and submultiples.

The first of these two essentials involves the dimensions of permeability. This seems to be purely a matter of convention. Many of us, by sheer force of habit and association with text books of thirty years ago, pay lip service to the idea of entirely unknown dimensions for both permittivity and permeability. But a careful examination of our actual mental process never seems to be quite in keeping with this idea. None of our everyday practical measurements makes either quantity other than a numeric and in the light of present day views of the constitution of matter there is equally no justification for the adoption of a convention leaving the dimensions of μ and κ uncertain. That being so, H and B are only alternative names for the same thing, *viz.*, the state of space giving rise to a mechanical force on the pole of a magnet. This attitude will certainly be objected to on the grounds that magnetizing force and flux density are manifestly different things. The answer to this objection, however, is that there is a very general confusion between magnetizing force and magnetic field strength, arising from the fact that often they are the same and generally they are denoted by the same letter, H. The confusion vanishes if magnetizing force is defined as that component of the magnetic field attributable to current-carrying coils or magnets external to the medium under discussion. Another objection to this attitude is that $\sqrt{\mu\ \kappa}$ is commonly regarded as having the dimensions of the reciprocal of a velocity. But according to the electromagnetic theory it is $\sqrt{\mu\ \kappa}/c$, where c is the ratio of the units, that has the dimensions of the reciprocal of a velocity, and it is at least as logical and far more reasonable to attribute the dimensions of velocity to c. For, though at first sight c is merely the ratio of two magnitudes it is in reality more than that, *viz.*, a relation between two different observable properties of the ether of space which in different ways may be used to measure a quantity of electricity. It is one of the peculiarities of the medium that the two states should lead to simply self-consistent measures of quantity and there is no difficulty in adopting a convention requiring that this relationship should have dimensions.

For these reasons, and mainly because they seem on examination to be the nearest approach to the actual views held by most physicists and engineers, the writer has for many years advocated the convention of B and H, both denoting field strength and μ being a numeric.

With regard to the second essential upon which a decision is required, the only reason for departing from the c. g. s. system seems to be that the practical and theoretical units should be made to coincide. But is not this advantage very much overrated? The practical units of today are not those of tomorrow, *e. g.*, volt and kilovolt. Moreover the range of magnitude over which electrical measurements are made is so wide that decimal multiples and sub-multiples for practical working are always inevitable. The advantage to be gained is therefore very slight and in view of the general adoption of the c. g. s. system for all theoretical work it seems almost impossible that the great inconvenience (not to say cost) of the change could ever be justified. Anyone who has had experience of the slow progress of the introduction of a decimal system of units into Great Britain will realize the colossal effort and powerful arguments that would be necessary to bring about a departure from the c. g. s. system. And so far as the writer can see, these arguments do not exist.

"The Future is greater than the Past," and this is not to say that no change will ever be made. As knowledge advances some stage may well be reached where a stock-taking and reorganization of the units is unavoidable. It is merely the writer's contention that this stage has not yet been reached. But in expressing this view he does not wish to suggest any failure to appreciate the great value of Dr. Kennelly's description of the various new systems that have been proposed up to date. A frequent examination of all these proposals is one of the essential necessities for the progress of science.

D. P. Savant: It is my opinion that the c. g. s. system should be left as it is at present. If a "practical" system is desired, then it seems to me that the m.-k.-s.-ohm would be preferable.

H. H. Race: There is one change in nomenclature which I believe would result in a simplification of present practise. We now use the terms:

resistance and resistivity
conductance and conductivity
reluctance and reluctivity
permeance and permeability
elastance and elastivity
capacitance and permittivity

In each case except the last the name of the quantity per unit volume has the same root as the quantity from which it is derived. The reason for the difference in the last case is that historically two names, capacitance and permittance, have both been used for the same quantity.

For simplicity we should use *either* "capacitance," "capacitor," and "capacitivity" *or* "permittance," "permittor," and "permittivity" and we *should not* use words from both sets.

It seems to me that the first set should be retained and the second set discarded for the following reasons:

1. The words capacitance and capacitor are being used generally in the literature and therefore the word capacitivity should find favor and ready adoption.
2. The word permittance is not used and therefore the word permittivity should be dropped in favor of the word capacitivity.
3. Both roots should not be used because of the confusion caused to students, teachers, and readers of technical literature who are not extremely well acquainted with the present nomenclature. This is especially true of foreigners who look for a different meaning in the two words and may confuse dielectric with magnetic phenomena because of the similarity between the words permeability and permittivity.

H. L. Curtis: This paper gives a very excellent résumé of the historical development of electrical and magnetic units. It points out the unsatisfactory conditions of the magnetic units at the present time and invites discussion as to the best system of units to adopt for international use.

In my opinion the Giorgi system has more disadvantages than advantages. It uses the international ohm as one of the basic units. However, the international ohm does not have the same definiteness as do the units of mass, length, and time. Any attempt to make it definite will probably involve the establishment of a denite value for the permeability of space. It therefore seems better to base the system on the permeability of space rather than on a derived unit which assumes the constancy of this quantity.

I agree with Professor Kennelly that the first step in establishing the units is to agree on the definitions of the magnetic quantities. The present stumbling block seems to be in determining whether magnetizing force and magnetic induction are the same physical entity and hence should be defined in terms of the same unit. That they are different theoretical entities seems to me to be shown by the fact that in a crystalline material the two vector fields which represent these quantities may not have the same direction. Hence it seems to me essential to distinguish between these two quantities in establishing a system of units.

It does not seem unnatural to me that the magnetic units which are derived from the practical electrical units are not of convenient magnitude for general use. It does not seem possible to establish any extensive system of units which are based on fundamental physical laws in which all of these units will be equally convenient for practical use. This condition exists in the strictly mechanical units where the fundamental units centimeter, gram, and second are of convenient size but the derived unit of work, the erg, is too small to be of practical use so that a multiple, the joule, is commonly employed.

I note several inconsistencies in the abbreviations for square centimeter and cubic centimeter in Table II.

Leigh Page: (communicated after adjournment) The subject of electromagnetic units presents a situation which has become more and more complicated the further science has developed. At the present time three systems of c. g. s. units are in common use: the electrostatic system (e. s. u.), the electromagnetic system (e. m. u.) and the Heaviside-Lorentz system (h. l. u.). The first two are used generally by the experimenter, e. s. u. being employed if his problem is one involving electrostatic phenomena, or e. m. u. if he is working with magnetic substances, currents, or induced electromotive forces. On the other hand the h. l. u. system is employed in nearly all recent papers and text-books on electromagnetic theory both in the United States and abroad. Finally the electrical engineer uses a system of practical units (q. e. s.) obtained from the electromagnetic system by employing the *earth-quadrant* $\{(10)^9 \text{ cm.}\}$ as unit of length and the *eleventh-gram* $\{(10)^{-11} \text{ gm.}\}$ as unit of mass, the *second* being retained as the unit of time. Nearly all electrical instruments are calibrated in q. e. s.

The seriousness of the situation lies in the fact that the fundamental equations of the science are in many cases different according as one or another system of c. g. s. units is employed. Thus the student has to learn that the force in dynes between two charges q and q' at a distance r apart is

$$F = \frac{q\,q'}{r^2} \text{ in e. s. u.}$$

$$F = c^2 \frac{q\,q'}{r^2} \text{ in e. m. u.}$$

$$F = \frac{q\,q'}{4\,\pi\,r^2} \text{ in h. l. u.}$$

Fortunately the practical system (q. e. s.) as given by Maxwell[3] does not introduce a new set of equations, the equations used being identical in all cases with those expressed in e. m. u. The practical units differ from the electromagnetic units solely in the choice of a larger unit of distance and a smaller unit of mass. The practical units which have received international approval are the *joule*, *watt*, *volt*, *ohm*, *ampere*, *coulomb*, *farad*, and *henry*. I shall refer to these eight as the *recognized* practical units.

Recently Giorgi, and Dellinger and Bennett[4] have proposed new systems of units which preserve the eight recognized practical units mentioned above but which differ from the q. e. s. system in that they change the form of many of the fundamental equations. If either of these systems should be adopted by the electrical engineer the student would have to memorize *four* sets of fundamental equations instead of *three* as at present. Again the Giorgi and Dellinger-Bennett systems make the specific inductive capacity and the permeability of empty space dimensional constants differing greatly in magnitude from unity. For this there is absolutely no warrant in our present theory, which leads unambiguously to such expressions as $B = H + 4\,\pi\,I$ (e. m. u.), showing at once that B, H, and I are all to be expressed in the same units and that μ is a pure number having the value unity for empty space. To attribute magnitude and physical dimensions to the permeability of empty space replaces "economy of thought" by profligacy of thought. It is about as reasonable as to write the fundamental equation of dynamics in the form

$$F = k\,m\,a$$

where k is some mysterious dimensional constant of magnitude $(10)^9/4\,\pi$ (or any other value you choose) which makes force a quite different thing from the product of mass by acceleration. Furthermore it is important in this connection not to lose sight of the fact that B inside a permeable medium is the mean H, and that the quantity generally known as the magnetic field intensity or magnetizing force is really not the H actually existing in the medium at all, but the purely fictitious field which would exist if each amperian circuit were replaced by an equivalent magnetic shell.

The adoption of either the Giorgi or Dellinger-Bennett system of units would make a bad situation much worse for anyone whose interest covered the whole field of electromagnetism. Even for those whose attention is limited to the field of circuit-theory alone, such small advantages as the inclusion of the commonly used volt/cm. as a unit in the Dellinger-Bennett system are acquired at the expense of a meaningless dimensional constant for the permeability of empty space. It is a simple matter to show that Maxwell's (q. e. s.) system is the only one which can preserve the eight recognized practical units and at the same time preserve the form of *all* the equations of electromagnetism as deduced in electromagnetic units.

The object of this contribution is to propose two systems of units, one c. g. s. and the other practical, which are free from the objections mentioned above and which should satisfy the requirements of theorist, experimenter, and engineer. The proposed systems are based on the following criteria:

1. A *single* set of equations shall suffice for both proposed systems of units. If an equation is deduced in c. g. s. units the same equation shall hold in practical units and *vice versa*. Thus the student need learn only a single set of equations instead of three sets as at present, or four sets as would be required if the Dellinger-Bennett system of units should be adopted in place of Maxwell's q. e. s. system.

2. The eight recognized practical units shall be retained in the proposed practical system. This requirement is essential on account of the general use of these units and the fact that nearly all electrical instruments are calibrated in terms of them.

3. The set of equations chosen shall exhibit the symmetry existing between electrical quantities on the one hand and magnetic quantities on the other.

The only system of units in general use at the present time which satisfies condition (3) is the h. l. u. system. Therefore we shall write our fundamental equations in the form:

$$F = \frac{q\,q'}{4\,\pi\,r^2} \qquad (1)$$

$$F = \frac{m\,m'}{4\,\pi\,r^2} \qquad (2)$$

$$\nabla \times \mathbf{H} = \frac{1}{c}(\dot{\mathbf{D}} + \mathbf{j}) \qquad (3)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\dot{\mathbf{B}} \qquad (4)$$

$$\mathbf{D} = \mathbf{E} + \mathbf{P} = k\,\mathbf{E} \qquad (5)$$

$$\mathbf{B} = \mathbf{H} + \mathbf{I} = \mu\,\mathbf{H} \qquad (6)$$

$$\mathbf{F} = q\left[\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B}\right] \qquad (7)$$

$$V = \int \mathbf{E}\,.\,d\,\mathbf{l} \qquad (8)$$

$$i = j\,A = \frac{q}{t} \qquad (9)$$

$$R = \frac{V}{i} \qquad (10)$$

$$C = \frac{q}{V} \qquad (11)$$

$$L = -\frac{V}{\dfrac{d\,i}{d\,t}}, \qquad (12)$$

choosing as the system of c. g. s. units to be employed the h.l.u. units advocated by Heaviside and Lorentz and generally used by writers on electromagnetic theory. The symbols appearing in these equations have their usual significance, c being the velocity of light, V potential or e. m. f., j current density, and A cross-section.

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[3] "Electricity and Magnetism," Vol. II, p. 268.

[4] A. E. Kennelly, *Magnetic Circuit Units*, A. I. E. E. Proc., Jan. 1930.

The problem remaining consists in investigating the possibility of choosing new units of time, length, and mass so as to obtain a practical system employing the *same fundamental equations* in which the units of energy, power, e. m. f., resistance, current, charge, capacitance, and inductance shall be the eight internationally recognized units. Evidently it is necessary in so far as the pairs energy-power and charge-current are concerned that we preserve the *second* as the unit of time. Therefore we may introduce new units of length and mass only. Denoting h. l. u. by letters without subscripts and units in the proposed practical system by the same letters with the subscript p, we have

$[t_p] = [t] = 1$ sec., $[l_p] = \alpha\,[l] = \alpha$ cm., $[M_p] = \beta\,[M] = \beta$ gm.

We shall use a similar notation for the numerics associated with each physical quantity, using a letter without subscript in the case of h. l. u. and with the subscript p in the case of the proposed practical units. To determine the two constants α and β we need two equations. Let us consider energy and charge. Then

$$U\left[\frac{M\,l^2}{t^2}\right] = U_p\left[\frac{M_p\,l_p^2}{t_p^2}\right] = \alpha^2\beta\,U_p\left[\frac{M\,l^2}{t^2}\right],$$

$$U_p = \frac{1}{\alpha^2\,\beta}\,U.$$

But
$$U_p = (10)^{-7}\,U$$
in order that the *joule* shall be the unit of energy in the practical system. Therefore

$$\alpha^2\beta = (10)^7. \tag{1}$$

Similarly for charge

$$q\left[\frac{M^{1/2}\,l^{3/2}}{t}\right] = q_p\left[\frac{M_p^{1/2}\,l_p^{3/2}}{t_p}\right] = \alpha^{3/2}\beta^{1/2}q_p\left[\frac{M^{1/2}\,l^{3/2}}{t}\right]$$

$$q_p = \frac{1}{\alpha^{3/2}\ \beta^{1/2}}\,q,$$

and if the *coulomb* is to be the practical unit of charge

$$q_p = \frac{1}{\epsilon}\,(10)^{-10}\,q$$

where ϵ is the number

$$\epsilon = 0.299796\,\sqrt{4\,\pi} = 1.06275$$

Hence

$$\alpha^3\beta = \epsilon^2\,(10)^{20}. \tag{2}$$

Solving (**1**) and (**2**) for α and β

$$\alpha = \epsilon^2\,(10)^{13}, \qquad \beta = \frac{1}{\epsilon^4}\,(10)^{-19}. \tag{3}$$

The numbers α and β being known, the six remaining practical units may be determined. Evidently the units of power and current are the *watt* and the *ampere* respectively, since the *second* has been retained as the unit of time. Employing the method used above for the other units, the units of e. m. f., resistance, capacitance, and inductance are found to be the *volt, ohm, farad,* and *henry* respectively. Therefore requirement (**2**) is satisfied. In this connection it should be noted that inductance is defined as the e. m. f. per unit time rate of decrease of current and not as the flux of induction per unit current. The two are not equivalent in the system of fundamental equations which have been chosen, e. m. f. being the $(1/c)$th part of the time rate of decrease of flux.

We conclude with a table of the more important equivalents in h. l. u. and the proposed practical units, giving the unit in parentheses in cases where it has received a name. The quantities listed are the numerics in each case. The ratio of the units is the reciprocal of the ratio of the numerics.

		Proposed practical	H. L. U.
Time	t_p	(sec)	$=\ t$ (sec)
Length	l_p		$= \epsilon^{-2}\,(10)^{-13}\,l$ (cm.)
Mass	M_p		$= \epsilon^4\,(10)^{19}\,M$ (gm.)
Force	F_p		$= \epsilon^2\,(10)^6\,F$ (dyne)
Energy	U_p	(joule)	$= (10)^{-7}\,U$ (erg)
Power	P_p	(watt)	$= (10)^{-7}\,P$ (erg/sec.)
Charge	q_p	(coulomb)	$= \epsilon^{-1}\,(10)^{-10}\,q$
Elect. Pot. or e. m. f.	V_p	(volt)	$= \epsilon\,(10)^3\,V$
Elect. Int., Elect. Disp. and polarization	E_p		$= \epsilon^3\,(10)^{16}\,E$
Capacitance	C_p	(farad)	$= \epsilon^{-2}\,(10)^{-13}\,C$ (cm.)
Current	i_p	(ampere)	$= \epsilon^{-1}\,(10)^{-10}\,i$
Resistance	R_p	(ohm)	$= \epsilon^2\,(10)^{13}\,R$ (sec./cm.)
Pole strength	m_p		$= \epsilon^{-1}\,(10)^{-10}\,m$
Mag. pot. or m. m. f.	Ω_p		$= \epsilon\,(10)^3\,\Omega$
Mag. Int., Mag. Ind. and Int. of of Mag	H_p		$= \epsilon^3\,(10)^{16}\,H$
Inductance	L_p	(henry)	$= \epsilon^2\,(10)^{13}L$ (sec.^{2}/cm.)

The velocity of light in the proposed practical units is

$$c_p = \epsilon^{-2}(10)^{-13}\,c = 0.00265438 = \frac{1}{376.735}$$

and the powers of ϵ appearing in the electrical quantities in the table are

$$\epsilon = 1.06275 = \frac{1}{0.940955}$$

$$\epsilon^2 = 1.12944 = \frac{1}{0.885396}$$

$$\epsilon^3 = 1.20031 = \frac{1}{0.833118}$$

The specific inductive capacity k and the permeability μ, being pure numbers, have the same values in the proposed practical units as in h. l. u. The first is the same as in e. s. u. and the second is the same as in e. m. u.

We shall consider now some of the objections which may be raised to the scheme proposed. The most serious of these is that some of the fundamental equations as well as many of the ratios of the c. g. s. to the proposed practical units demand a knowledge of the velocity of light. This, however, is a characteristic of any *complete* set of equations, whether we use e. m. u. or any other system of units. Furthermore, in view of Michelson's extremely accurate determination of the velocity of light this objection is not likely to lead to changes in the ratios comparable with the errors of even the most precise electrical measurements for a great many years to come.

Next Ampère's law and Faraday's law in the proposed units involve the constant factor $(1/c)$. This factor, however, is no more troublesome in numerical calculation than the factors $4\,\pi$ and $(10)^{-8}$ which the engineer is accustomed to use at the present time. In fact, the proposed equations have the advantage that only *one* factor need be learned, instead of *two* as at present. On the other hand the proposed scheme does not alter the form of the Equations (**10**), (**11**), (**12**) which are the ones perhaps most used by the engineer.

The plan proposed, then, has the following very vital advantages over present usage. (1) The equations are in symmetrical form. This results in halving the number of relations between practical and c. g. s. units. Thus the ratio of E to E_p is the same as that of H to H_p, and so on. (2) Only *one* set of equations need be learned, instead of *three* as at present or *four* if the Giorgi or Dellinger-Bennett system were adopted. The quantities k and μ are pure ratios having the same values in c. g. s. and practical units, and the indefensible introduction of dimensional constants of magnitude far from unity for k and μ of empty space is avoided. (3) The eight internationally recognized practical units are retained unaltered in the proposed scheme.

L. W. McKeehan: Professor Kennelly points out that a decision regarding the likeness or unlikeness of the quantities H and B is necessary before any detailed recommendation regarding a system of magnetic units can be made. I wish to show that there is no physical basis for supposing H and B to be different in kind. If, then, as a matter of convenience it is still desired to retain both symbols, the quantities they represent should, it seems clear, be measured in terms of the same unit. The least disturbance of existing practise will probably result if this unit is called the *gauss*.[5]

Ideas regarding the constitution of matter have changed a good deal since the time of Maxwell, when the subject under discussion first arose in its present form. Even as late as 1900 it was still allowable to regard matter as essentially continuous, and to speak without apology about many spatial properties of matter, such as density, electrical resistivity, and magnetic permeability. It was even allowable to attribute such properties to space itself (then called the luminiferous ether). When we use these terms nowadays we are ignoring the whole of modern physics. The present physical hypothesis regarding matter is that it consists of continuous distributions of positive and negative electricity in relative motion. The elementary electrical charges, electrons and protons, of which matter is still imagined to be built, have, to be sure, lost the definite boundaries it was, until recently, customary to imagine about them, but the electric charge density within the space which matter "occupies" is still supposed to vary enormously from point to point. Different kinds of matter differ, on this hypothesis, only in the arrangement, amount, and relative velocity of these distributed electric charges.

Now if we knew completely the motion of electricity everywhere in space we could, at least formally, calculate a value of H at every point, whether the point was, as we have been accustomed to say, in "empty space" or in "matter." We should notice at once that in "empty space" the values of H would not vary rapidly as we changed the place, or the time, for which the calculation was made. In "matter" on the other hand it would make a great difference in H to change the place considered by as little as 10^{-9} centimeter, or to change the time considered by as little as 10^{-15} second, to take reasonably large changes, on atomic scales, in both cases.

If we decide, in view of this fine-grained structure of H, to consider only its average value over such large spaces and such long times as may be necessary to get nearly constant values in "matter" as well as in "empty space," we will find that this average value is precisely the value of B which a believer in continuous matter would have to postulate to explain the results of ordinary (*i. e.*, large-scale and slow) measurements.

In order to support this statement, which may be very surprising to many engineers, let us conduct an imaginary experiment, as follows. Select a region in which H has the constant value H_o, and in which, therefore, there is no matter already present to interfere with further operations. Into this region introduce, one at a time, iron atoms, each consisting of electricity of both signs in relative motion, and each capable of producing a non-uniform magnetic field of consequence in the space it occupies and in the immediate neighborhood. The average value of H in a small volume including our first iron atom is different from what it was before, because an atom of iron has that special kind of motion of its electrical charges which we mean when we say an atom is paramagnetic.[6] The average value will be greater than H_o if we place the iron atom so that the intense part of its local magnetic field agrees in direction with H_o, and will be less than H_o if we reverse the orientation of the atom with respect to H_o. The average value of H in the whole volume at our disposal is affected in the same sense as in the small volume by the introduction of the first atom, but, of course, to a smaller degree.

Let us now arbitrarily decide to put all the atoms into places iron atoms might occupy in a bar of iron of any convenient size, and to place them all in the right way to raise the average value of H in the space thus filled. Let the average value so attained, when all are in place, be H'. The bar of iron thus created is easily seen to be magnetically saturated in the original direction of H_o.

Let us repeat the experiment from the beginning, first changing external conditions in such a way that the original field has the value $-H_o$, and taking care to put in every atom in such a way as to make the average H within our new bar of iron more and more negative, finally attaining the value, $-H'$, corresponding to negative saturation.

Let us now repeat both the processes just described, but let us do at one stroke the unbuilding of bar number one, the reversal of H_o and the building of bar number two, and watch what happens in a search coil, surrounding the middle of the space where the bars are to be annihilated and created, and connected to a ballistic galvanometer. This measuring device records, of course, just what it would if the search coil surrounded a vacuum of the dimensions of the bar, in which we reversed a magnetic field of intensity H'. (The throw of the ballistic galvanometer is proportional to the surface integral of the normal component of H' over the area bounded by the search coil.) But we have done exactly the same thing in our imaginary processes as can actually be done with a single bar of iron by reversing H_o, if H_o is chosen high enough to keep the real iron bar magnetically saturated. In this more feasible experiment the quantity we would expect to measure with search coil and galvanometer would be what we usually call the magnetic induction, *i. e.*, B. (The throw of the ballistic galvanometer is proportional to the surface integral of the normal component of B over the area bounded by the search coil.) We have therefore proved that $B = H'$, and H' by hypothesis was the average value of H within the bar, including the effects of the atoms present within its boundaries.

I have pointed out in another place (*Optical Society of America Jl.* and *Rev. of Scientific Instruments 19*, 1929, 213-242) that whether we make measurements by artificial electric circuits or by those natural electric circuits which are found in ferromagnetic metals, we measure the average magnetic condition in a relatively extensive space and for a relatively long time, *i. e.*, we measure B or its changes. If I am right in thinking that sub-microscopic H and average H, or B, only need to be distinguished because our senses or instruments are not acute enough to perceive their common origin, I am, of course, also right in advocating a single unit for both and in denying dimensions to the factor μ in the equation $B = \mu H$.

The equation $B = \mu H$, by the way, is an extremely awkward equation in some respects. The factor μ assumes all values between $+\infty$ and $-\infty$ for one and the same specimen of any ferromagnetic material as we traverse a hysteresis loop. The effort to restrict what we may be allowed to do to H (and B), so as to limit this erratic behavior of μ, and make it useful in the description of materials, is interesting but would carry us too far into the subject of magnetic testing. The equation $B = H + 4\pi I$ upon which $B = \mu H$ may be considered to depend, has the advantage, by comparison, that I has a limited range of values and that the maximum value of I seems to depend more simply upon the nature of the magnetic material than does any other magnetic vector. The disadvantage of the equation $B = H + 4\pi I$ is that H appearing in it is a fictitious magnetic field due partly to completely external electric currents and partly to the motion of electricity in the non-adjacent regions of the piece of matter under investigation.

The magnetization, I, is of course an average quantity just as much as B is, so I am not defending its use as necessary on physical grounds. Pole strength and magnetic moment are other average quantities that are only useful if we know when to abandon them. Regardless of how carefully we may define and name such properties and suitable units for expressing their values, we need not be surprised if they quietly drop out of use in due time. Meanwhile I suggest caution in sponsoring too many of these temporary concepts.

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[5] The plural of *gauss* may be either *gausses* or *gauss*. I prefer the latter as more euphonious.

[6] This is a deliberate circumlocution for the statement: "An atom of iron has a magnetic moment different from zero."

Much of the confusion that now exists regarding magnetic units arises from forcing the analogy, pointed out by Du Bois, between electric and magnetic circuits. That the analogy is imperfect is plain from the fact that the maintenance of a steady electric current, in a circuit composed of imperfect conductors of electricity, requires a continuous expenditure of energy, whereas the maintenance of a steady magnetic current, in a circuit composed of imperfect conductors of magnetism, does not.

Another word of caution may be in order on devising names for the reciprocals of useful units. The physicists and chemists have long been using the phrase "per gram" as part of the names of many compound units but I have never heard it proposed to substitute for this phrase the word "marg." Too much consistency in nomenclature may be as bad as too little, if we merely succeed in adding items to the long list of things that everyone must succeed in forgetting as soon as he hears them. Similarly, it does not do to be too particular about having a fundamental unit of a "convenient" size, as convenience is not an absolute property. The student of atomic properties has more reason than the electrical engineer to criticize the c. g. s. system for not furnishing him with convenient units of mass, length, or time. Nevertheless, he continues without complaint to use the c. g. s. system for theoretical work and to accept in his results as many powers of ten as may be necessary (10^{65} appears in a recent paper). The units used in expressing the results of routine work need not, of course, be basic units in any system. Finally, if anybody is much confused by the factor 4 π which crops up here and there in the c. g. s. system and in the various substitutes mentioned in this paper, he may adopt Heaviside-Lorentz units, as many theoretical physicists have already done.

A. E. Kennelly: The interesting discussion with which the paper has been favored has clearly revealed how serious are the differences of opinion among us as to the definitions and interrelations of the terms used to describe the magnetic circuit, and how great is the need for standardization in this field, both nationally and internationally. It is evident that before we can hope to accomplish any useful constructive work on magnetic-unit terminology, we must first seek agreement on the definitions of certain fundamental terms, especially as to magnetizing force H, magnetomotive force $\mathfrak{F}$, magnetic flux ϕ, permeance $\mathcal{P}$, and specific permeability μ.

It has been shown in the discussion that the symbol H is used with two meanings, namely:

1. Gradient of m. m. f. as expressible in gilberts per cm., or ampere-turns/cm.
2. The magnetic field produced in free space or nonmagnetic material by the gradient of m. m. f. or gilberts cm.

Here No. 1 is the cause, and No. 2 is the effect. No. 1 can be determined, in simple cases, from a knowledge of the ampere-turns linked with the magnetic circuit and the mean length of the same; whereas No. 2 is associated with a resulting state of stress in the magnetic circuit, detectable by iron filings, compass needles, or a twistable search coil. If we can secure agreement upon limiting the use of H to gradient of m. m. f., under the name of *magnetizing force*, and requiring that all fields shall be designated by B with suitable subscripts, such as B_o for the field in free space, it would seem that agreement may be facilitated. We then have in circuits of nonmagnetic material:

$$B_o = \mu_o H \qquad \text{(maxwells per cm.}^2\text{)}$$

where μ_o is *space permeability*. In the c. g. s. magnetic system, the numerical value of μ_o is taken as unity. The question is however, whether μ_o is a mere unity number, or whether it has physical dimensions. If it is a mere number, then B_o is physically identical with H, and both may be expressed in *gilberts per cm.* In that case, the *maxwell*, as a unit of flux ϕ, becomes identical with a *gilbert-cm.*, and it becomes proper to express magnetic fluxes in *gilbert-cm.* The name *maxwell* for the unit of flux ϕ becomes not erroneous but superfluous. We thus have the alternative of taking μ_o as dimensionless, with flux density B_o expressible in *gilberts per cm.*, and flux ϕ expressible in *gilbert-cm.*, or of taking μ_o as having dimensions, and keeping the *maxwell* as a necessary name for flux, together with the *maxwell per cm.*^{2} a necessary name for flux density B.

There seems to be more likelihood of securing agreement on dimensions of some kind for μ_o, with necessary retention of the *maxwell*, than for a purely numerical value for μ_o and no need for the *maxwell*.

It seems to be generally admitted, however, that *relative permeability* or *specific permeability*, *i. e.*, the ratio of flux density B in a magnetic material, such as iron, to the flux density B_o in free space under the same magnetizing force, is a mere numeric and therefore does not call for a unit name. Thus, specific permeability is generally accepted as

$$\mu = \frac{\text{flux density in medium}}{\text{flux density in free space}} = \frac{B}{B_o} \qquad \text{(numeric)}$$

The *absolute permeability* of a magnetic material, at a specified value of magnetizing force H, would then be the ratio of the flux density produced in the material to the exciting magnetizing force H, or

$$\mu\,\mu_o = \frac{\text{flux density in the medium}}{\text{magnetizing force}}$$

$$= \frac{B}{H}\left(\frac{\text{gausses}}{\text{gilberts per cm.}}\right)$$

The precise dimensions of absolute permeability, as regards length, mass, and time, have not yet been ascertained.

In any case, we must agitate for discussion and analysis in all countries, in order to enable international agreement to be reached on some logical basis.